

# Using hyperbolic large-margin classifiers for biological link prediction

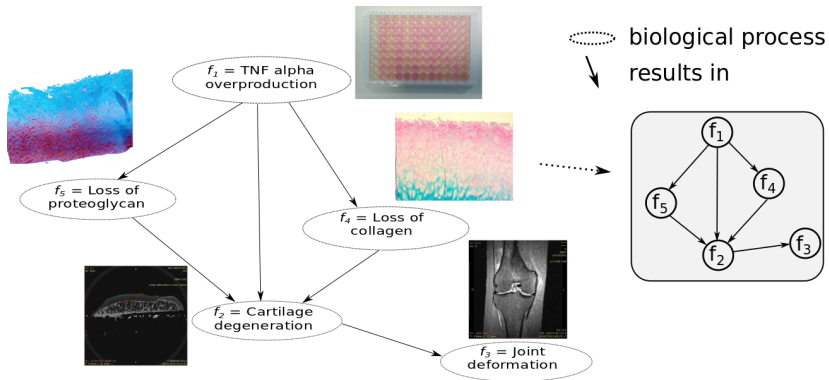
SemDeep-5 @ IJCAI 2019

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Vienna, Austria

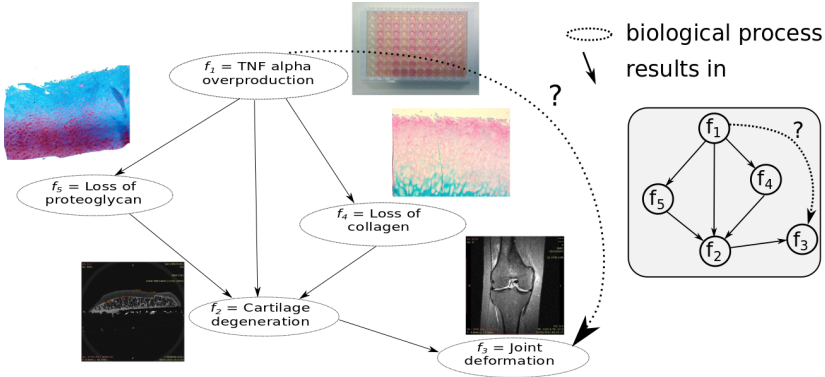
Aug 12, 2019

# Representing biological knowledge<sup>1</sup>



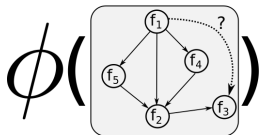
<sup>1</sup>"Shared hypothesis testing", Agibetov et al., J. Biomed. Sem., 2018

# Biological link prediction

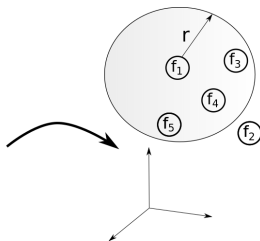


# Link prediction as distance based inference in embedding space

How to learn  $\phi$  ?



Graph domain



Embedding space  $\mathbb{R}^n$

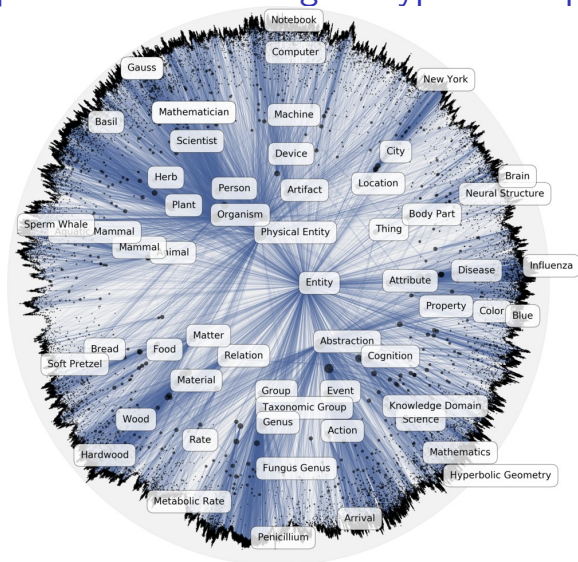
inference in embedding space based on distance

Points within  $r$ -ball centered around fixed point are connected in the graph domain

$$d(\phi(f_1), \phi(f_3)) \leq r \Rightarrow$$

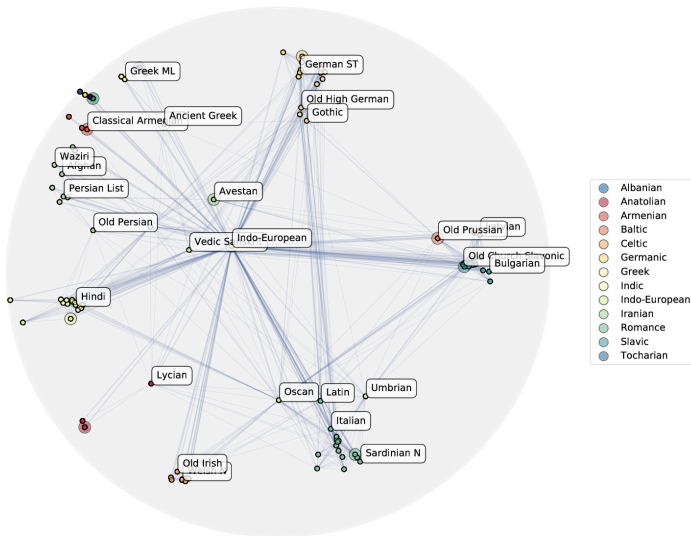
$f_1$  and  $f_3$  have a link

# Representation learning in Hyperbolic space

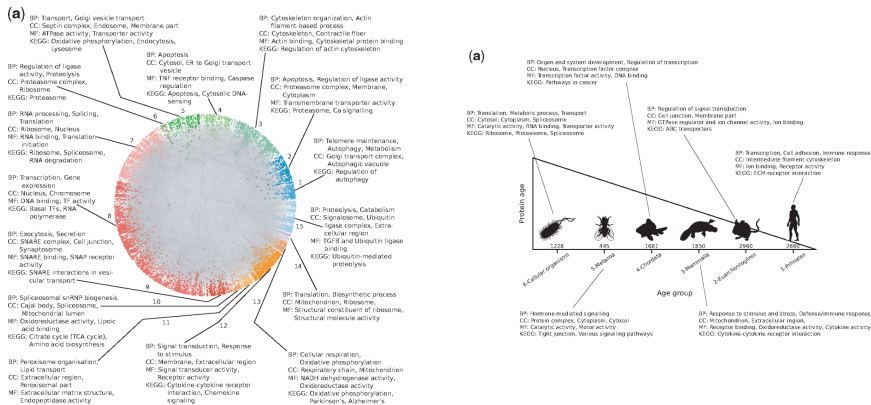


Nickel and Kiela. NIPS 2017

# Hierarchical relationship from hyperbolic embeddings



# Clusters of proteins and age groups from hyperbolic coordinates



Lobato et al. Bioinformatics 2018

# Hyperbolic embeddings

Same as in Euclidean case we try to learn a *link estimator*  
 $Q(u, v) \mapsto [0, 1]$  ( $u, v$  node pairs) with MLE

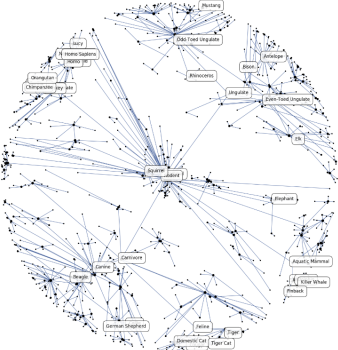
- ▶  $Pr(G) = \prod_{(u,v) \in E_{train}} Q(u, v) \prod_{(u,v) \notin E_{train}} 1 - Q(u, v)$
- ▶ If  $Q$  perfect estimator then  $Pr(x) = 1$  iff  $x = G$  (i.e., graph can be fully reconstructed)

Embeddings are parameters  $\Theta$  of link estimator  $Q$ ; trained with cross-entropy loss  $\mathcal{L}$  and negative sampling

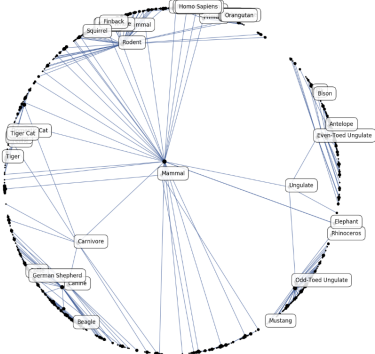
- ▶  $\mathcal{L}(\Theta) = \sum_{(u,v)} \log \frac{e^{-d(u,v)}}{\sum_{v' \in neg(u)} e^{-d(u,v' )}}$
- ▶ **But we perform all computations in hyperbolic space**



# Backpropagation to learn embeddings



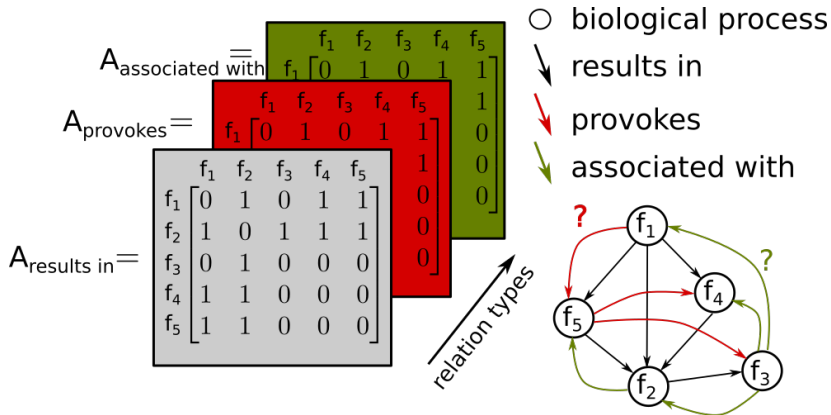
(a) Intermediate embedding after 20 epochs



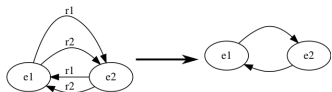
(b) Embedding after convergence

Nickel and Kiela. NIPS 2017

# Link prediction for multi-relational biological knowledge graphs



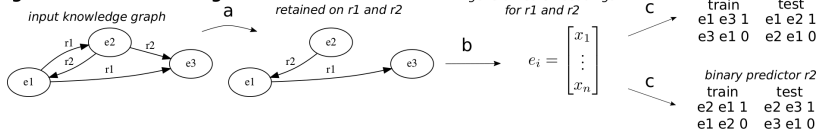
# Flattenning knowledge graphs <sup>2</sup>



Turn KG into unlabelled directed graph, s.t., no pair of nodes is connected with more than one arc (directed edge)

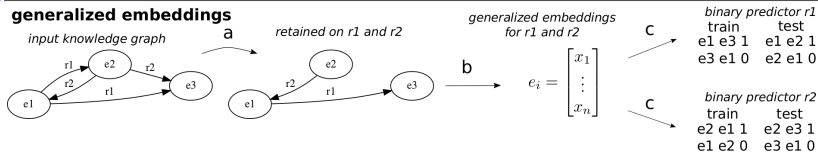
Dataset	# pairs connected with > 1 relation types
WN11	124/93003 (0.133%)
FB15-237	23700/310116 (7.642%)
UMLS	1343/6527 (20.576%)
BIO-KG	0/1619239 (0%)

## generalized embeddings

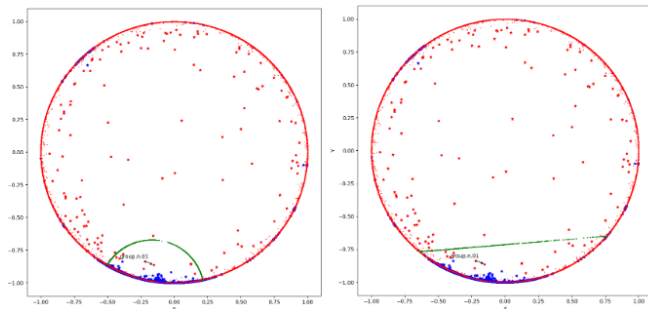


Compensate reduced information with binary classifiers fine tuned for each relation type

# Hyperbolic Large-Margin classifier (SVM)



Agibetov, Samwald. SemDeep-4@ISWC 2018



Cho et al. arxiv 2018

# Performance evaluation

Dataset	# relation types	# entities	max # links per relation type	min # links per relation type	mean # links per relation type	# pairs connected with > 1 relation types
UMLS	46	137	1021	1	142	1343/6527 (20.576%)
BIO-KG	9	346225	554366	6159	179915	0/1619239 (0%)

	Euclidean embeddings			Hyperbolic embeddings	
	dim $d$	Euc SVM	Hyp SVM	Euc SVM	Hyp SVM
UMLS	2	0.661 $\pm$ 0.023	0.616 $\pm$ 0.019	0.695 $\pm$ 0.026	<b>0.703 <math>\pm</math> 0.018</b>
	5	<b>0.780 <math>\pm</math> 0.023</b>	0.743 $\pm$ 0.024	0.735 $\pm$ 0.030	0.743 $\pm$ 0.024
	10	<b>0.793 <math>\pm</math> 0.025</b>	0.754 $\pm$ 0.022	0.767 $\pm$ 0.031	0.742 $\pm$ 0.026
BIO-KG	2	<b>0.692 <math>\pm</math> 0.010</b>	0.691 $\pm$ 0.010	0.613 $\pm$ 0.006	0.676 $\pm$ 0.009
	5	<b>0.776 <math>\pm</math> 0.010</b>	0.771 $\pm$ 0.011	0.697 $\pm$ 0.008	0.756 $\pm$ 0.011
	10	0.732 $\pm$ 0.009	0.723 $\pm$ 0.008	0.711 $\pm$ 0.010	<b>0.763 <math>\pm</math> 0.010</b>

# Lessons learned

## Benefit of learning hyperbolic embeddings

- ▶ fewer dimensions to capture latent semantic and hierarchical information
- ▶ scalability and interpretability (easier to visualize 2 or 3 dimensions)

## From our preliminary results

- ▶ hyperbolic embeddings learn hierarchical relationships in UMLS better than Euclidean embeddings (lower dimensions)
- ▶ For complex and big graphs (BIO-KG) train hyperbolic embeddings for longer periods ( $> 500$  epochs)

## Open issues and future directions

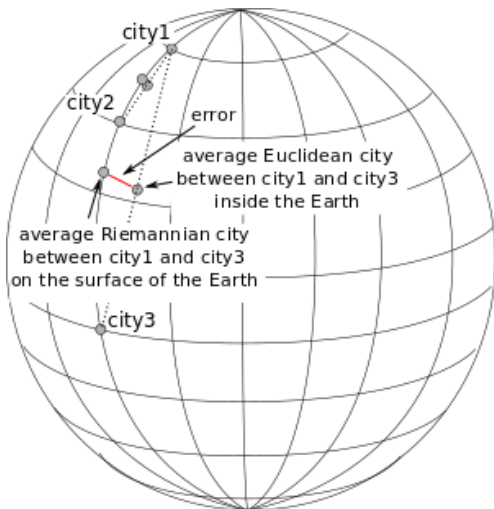
- ▶ even with recent advances in Riemannian SGD optimization <sup>3</sup>, learning hyperbolic embeddings still much slower than in the Euclidean case
- ▶ next steps should be focused on end-to-end hyperbolic embedding training (hyperbolic large-margin classifier loss is directly incorporated during the training process)
- ▶ code available at <https://github.com/plumdeq/hsvm>
- ▶ contact: [asan.agibetov@meduniwien.ac.at](mailto:asan.agibetov@meduniwien.ac.at)

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<sup>3</sup>“Gradient descent in hyperbolic space”. Wilson and Leimeister, 2018

# Why non-Euclidean space - (low-dim) manifolds

- ▶ Computing on a lower dimensional space leads to manipulating fewer degrees of freedom
- ▶ Non-linear degrees of freedom often make more intuitive sense
  - ▶ cities on the earth are better localized giving their longitude and latitude (2 dimensions)
  - ▶ instead of giving their position  $x, y, z$  in the Euclidean 3D space



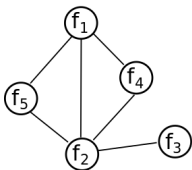


# Learning graph embeddings

- ▶ Learn *link estimate*  $Q(u, v) \mapsto [0, 1]$  ( $u, v$  node pairs) and approximate graph structure (connectivity) with MLE (maximum likelihood estimation)<sup>4</sup>

$$\text{▶ } Pr(G) = \prod_{(u,v) \in E_{train}} Q(u, v) \prod_{(u,v) \notin E_{train}} 1 - Q(u, v)$$

- ▶ If  $Q$  perfect estimator then  $Pr(x) = 1$  iff  $x = G$  (i.e., graph can be fully reconstructed)
- ▶  $Q$  can be *trained* to estimate links at different *orders*, i.e., approximate  $A^n$ .



1-order paths

$$A = \begin{matrix} & \begin{matrix} f_1 & f_2 & f_3 & f_4 & f_5 \end{matrix} \\ \begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{matrix} & \begin{bmatrix} 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 0 \end{bmatrix} \end{matrix}$$

2-order paths

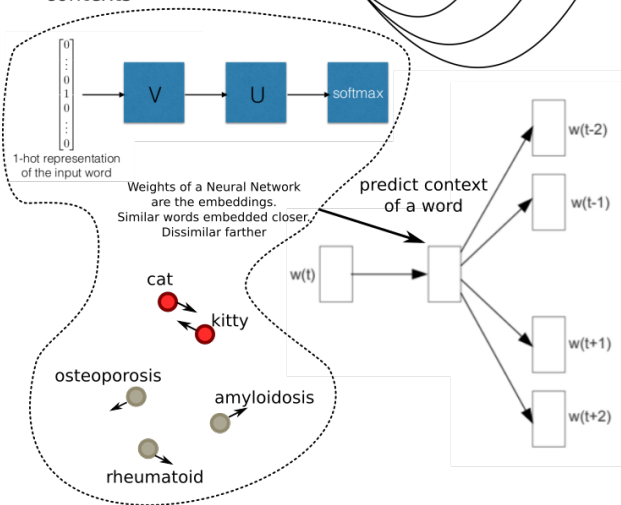
$$A(A) = A^2 = \begin{matrix} & \begin{matrix} f_1 & f_2 & f_3 & f_4 & f_5 \end{matrix} \\ \begin{matrix} f_1 \\ f_2 \\ f_3 \\ f_4 \\ f_5 \end{matrix} & \begin{bmatrix} 3 & 2 & 1 & 1 & 1 \\ 2 & 4 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 \\ 1 & 1 & 1 & 2 & 2 \\ 1 & 1 & 1 & 2 & 2 \end{bmatrix} \end{matrix}$$

<sup>4</sup>“Graph likelihood”, Haija, ... Perozzi, ..., CIKM17, NeurIPS 2018

# Similar principle as word2vec <sup>5</sup>

similar words  
will have similar  
contexts

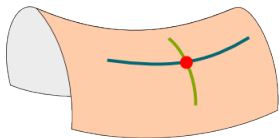
*cat ? kitty ? amyloidosis ?*  
"All of the sudden a     jumped from a tree to chase a mouse."



<sup>5</sup>“word2vec”, Mikolov et al., NIPS 2014

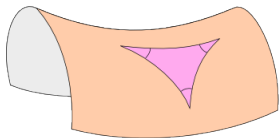
# What's so special about Riemannian geometry - curvature

Extremal directions curve  
in opposite directions



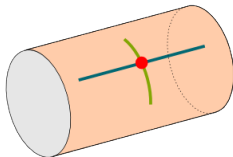
*Negative Curvature*

Triangle angles add up to  
**less** than  $180^\circ$



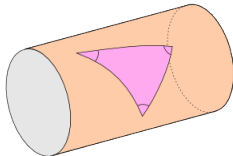
*Negative Curvature*

One extremal direction  
has zero curvature



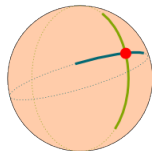
*Zero Curvature*

Triangle angles add up to  
**exactly**  $180^\circ$



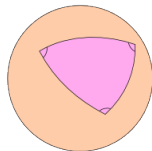
*Zero Curvature*

Extremal directions curve  
in the same directions



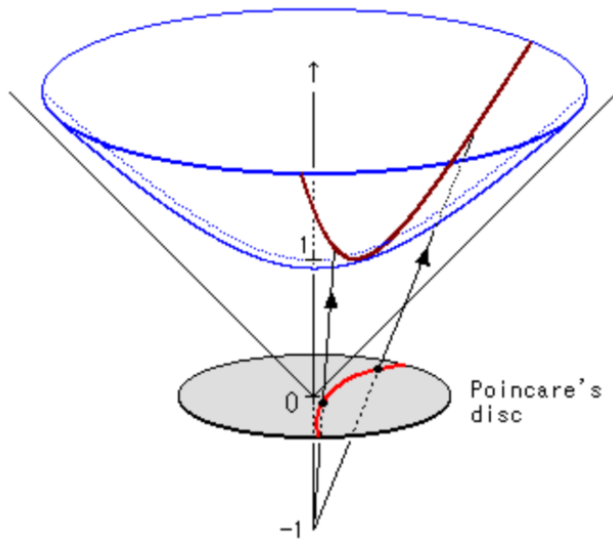
*Positive Curvature*

Triangle angles add up to  
**more** than  $180^\circ$

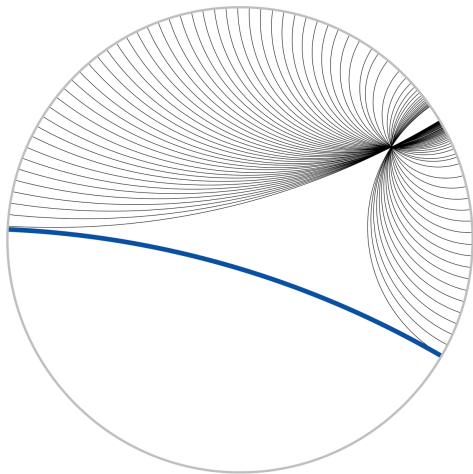
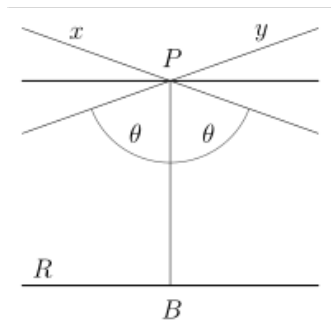


*Positive Curvature*

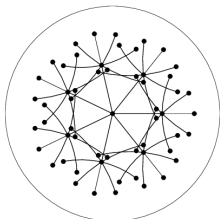
## Model of hyperbolic geometry



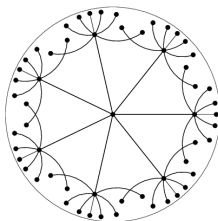
## Properties of hyperbolic geometry



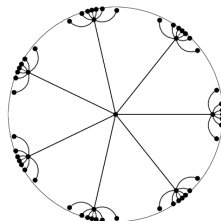
# Computing lengths in hyperbolic geometry



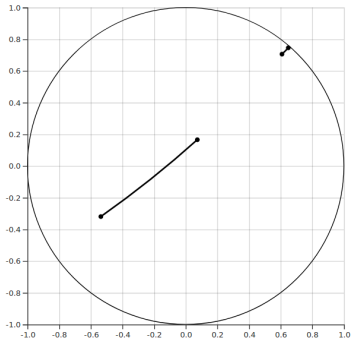
Radius 1



Radius 2



Radius 3



## Objects

ID: line-1  
Start: (0.649, 0.746)  
End: (0.609, 0.706)  
Hyperbolic Length: 1.791

ID: line-2  
Start: (0.074, 0.166)  
End: (-0.536, -0.319)  
Hyperbolic Length: 1.783

# Approximation of graph distance

