Polygon-filled circles

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1 Check line

Line: $\vec{L} = P_1, \vec{P_2}$
Target point (seed point + vector): $P_s + s \cdot \vec{v_s}$ where $|\vec{v_s}| = 1$.
Target point projected on line:

$$\begin{aligned} P_L &= P_1 + t(P_2 - P_1) = [t = (P_s + s\vec{v_s} - P_1)(P_2 - P_1)/|P_2 - P_1|^2] = \quad (1) \\ &= P_1 + \frac{(P_s + s\vec{v_s} - P_1)(P_2 - P_1)}{|P_2 - P_1|^2} \cdot (P_2 - P_1) \quad (2) \end{aligned}$$

Goal:

$$s = |P_s + s\vec{v_s} - P_L| \iff \quad (3)$$

$$s = \left|P_s + s\vec{v_s} - P_1 - \frac{(P_s + s\vec{v_s} - P_1)(P_2 - P_1)}{|P_2 - P_1|^2} \cdot (P_2 - P_1)\right| \iff \quad (4)$$

$$s^2 = \left|P_s + s\vec{v_s} - P_1 - \frac{(P_s + s\vec{v_s} - P_1)(P_2 - P_1)}{|P_2 - P_1|^2} \cdot (P_2 - P_1)\right|^2 \iff \quad (5)$$

$$s^2 = \left|P_s + s\vec{v_s} - P_1 - \left(\frac{(P_s - P_1)(P_2 - P_1)}{|P_2 - P_1|^2} + \frac{s\vec{v_s}(P_2 - P_1)}{|P_2 - P_1|^2}\right) \cdot (P_2 - P_1)\right|^2 \quad (6)$$

Let

$$\begin{aligned} P_1 &= (x_1, y_1) \\ P_2 &= (x_2, y_2) \\ P_s &= (x_s, y_s) \\ \vec{v_s} &= (x_v, y_v) \\ \alpha &= \frac{(P_s - P_1)(P_2 - P_1)}{|P_2 - P_1|^2} \quad \text{(i.e. a real number)} \\ \beta &= \frac{\vec{v_s}(P_2 - P_1)}{|P_2 - P_1|^2} \quad \text{(i.e. a real number)} \end{aligned}$$

Then

$$\begin{aligned}
s^2 = &\quad |P_s + s\vec{v_s} - P_1 - (\alpha + s \cdot \beta) \cdot (P_2 - P_1)|^2 && \iff \\
s^2 = &\quad |P_s + s\vec{v_s} - P_1 - \alpha(P_2 - P_1) - s \cdot \beta(P_2 - P_1)|^2 && \iff \\
s^2 = &\quad |P_s + s\vec{v_s} - P_1 - \alpha(P_2 - P_1) - s \cdot \beta(P_2 - P_1)|^2 && \iff \\
s^2 = &\quad (x_s + s \cdot x_v - x_1 - \alpha(x_2 - x_1) - s \cdot \beta(x_2 - x_1))^2 + \\
&\quad (y_s + s \cdot y_v - y_1 - \alpha(y_2 - y_1) - s \cdot \beta(y_2 - y_1))^2 && \iff \\
s^2 = &\quad (x_s - x_1 - \alpha(x_2 - x_1) + s(x_v - \beta(x_2 - x_1)))^2 + \\
&\quad (y_s - y_1 - \alpha(y_2 - y_1) + s(y_v - \beta(y_2 - y_1)))^2
\end{aligned}$$

Let

$$\begin{aligned}
\gamma_x &= x_s - x_1 - \alpha(x_2 - x_1) \\
\gamma_y &= y_s - y_1 - \alpha(y_2 - y_1) \\
\delta_x &= x_v - \beta(x_2 - x_1) \\
\delta_y &= y_v - \beta(y_2 - y_1)
\end{aligned}$$

Then

$$\begin{aligned}
s^2 &= (x_s - x_1 - \alpha(x_2 - x_1) + s(x_v - \beta(x_2 - x_1)))^2 + \\
&\quad (y_s - y_1 - \alpha(y_2 - y_1) + s(y_v - \beta(y_2 - y_1)))^2 \iff \\
s^2 &= (\gamma_x + s\delta_x)^2 + (\gamma_y + s\delta_y)^2 \iff \\
s^2 &= \gamma_x^2 + 2s\gamma_x\delta_x + s^2\delta_x^2 + \gamma_y^2 + 2s\gamma_y\delta_y + s^2\delta_y^2 \iff \\
0 &= s^2(1 - \delta_x^2 - \delta_y^2) - 2s(\gamma_x\delta_x + \gamma_y\delta_y) - \gamma_x^2 - \gamma_y^2 \iff \\
s &= \frac{2(\gamma_x\delta_x + \gamma_y\delta_y) \pm \sqrt{4(\gamma_x\delta_x + \gamma_y\delta_y)^2 - 4(1 - \delta_x^2 - \delta_y^2)(-\gamma_x^2 - \gamma_y^2)}}{2(1 - \delta_x^2 - \delta_y^2)}
\end{aligned}$$

Now we have the target point $P_s + s \cdot \vec{v_s}$ if the new circle will touch the line. To check whether it touches on the line or outside it calcultate $t = (P_s + s\vec{v_s} - P_1)(P_2 - P_1)/|P_2 - P_1|^2$. If $0 < t < 1$ then it is above the line. If $t < 0$ it is above P_1 and if $t > 1$ it is above P_2. If it is above any of the points, use formula for circles, but with $r_1 = 0$.

2 Check other circles

Circle: $P_1 = (x_1, y_1)$ with radius r_1 Target point (seed point + vector): $P_s + s \cdot \vec{v_s}$ where $|\vec{v_s}| = 1$.

Goal:

$$s = |P_s + s \cdot \vec{v_s} - P_1| - r_1 \tag{7}$$

Solution:

$$\begin{aligned}
(s+r_1)^2 &= |P_s + s \cdot \vec{v_s} - P_1|^2 \iff \\
(s+r_1)^2 &= |P_s - P_1 + s \cdot \vec{v_s}|^2 \iff \\
(s+r_1)^2 &= (x_s - x_1 + s \cdot x_v)^2 + (y_s - y_1 + s \cdot y_v)^2 \iff \\
s^2 + 2sr_1 + r_1^2 &= (x_s - x_1)^2 + 2s(x_s - x_1)x_v + s^2 x_v^2 + (y_s - y_1)^2 + 2s(y_s - y_1)y_v + s^2 y_v^2 \iff \\
0 &= s^2(1 - x_v^2 - y_v^2) + \\
& \quad 2s(r_1 - (x_s - x_1)x_v - (y_s - y_1)y_v) + \\
& \quad r_1^2 - (x_s - x_1)^2 - (y_s - y_1)^2 \iff
\end{aligned}$$

Let

$$\begin{aligned}
\alpha &= (x_s - x_1)^2 + (y_s - y_1)^2 \\
\beta_x &= (x_s - x_1)x_v \\
\beta_y &= (y_s - y_1)y_v
\end{aligned}$$

then (remember $|v_s| = 1$. i.e. $1 - x_v^2 - y_v^2 = 0$)

$$\begin{aligned}
0 &= 2s(r_1 - \beta_x - \beta_y) + r_1^2 - \alpha \\
s &= \frac{\alpha - r_1^2}{2(r_1 - \beta_x - \beta_y)}
\end{aligned}$$