Deduction

Deductive systems

Deductive databases

Parsing as deduction

Explaining deductive reasoning
DEDUCTIVE INFERENCEING

COMPUTATION

= 

DEDUCTION

+ 

CONTROL

Kowalski
LAYERS OF A DEDUCTION SYSTEM

Four layers connecting deduction and control

1. Logic – syntax and semantics of expressions
2. Calculus – syntactic derivations on formulas
3. Representation – state of formulas and derivations
4. Control – strategies and heuristics for selection

Most common examples

1. First-order predicate logic
2. Gentzen calculi  Resolution  Theory resolution
3. Tableau  Matrix  Clause graphs
4. Special techniques
PROPERTIES OF CALCULI

Function

*Syntactic derivation to verify semantic validity* (true under all interpretations)

Structure

Set of *axioms* (tautologies), minimal and *derivation rules*

Behavior

Forward chaining – deductive calculus
Backward chaining – test calculus

Assessing a calculus

*Soundness* – all axioms and derivable formulas are valid
*Completeness* – all valid formulas are derivable (in a finite number of steps)

Some fundamental insights

First-order predicate logic is *not decidable*, but *complete*
Higher-order predicate logic is *not complete* (Gödel 1931)
First-order predicate logic is the *most expressive* and still *complete* logic (Lindström 1969)
GENTZEN CALCULUS – NATURAL DEDUCTION

A positive deductive calculus

13 Derivation rules

\[
\begin{align*}
F & \quad G & \quad [F] & \quad Fa & \quad Fa & \quad [F] \\
F \land G & \quad F \lor G & \quad F & \quad F \Rightarrow G & \quad \forall x Fx & \quad \exists x Fx & \quad \neg F \\
\end{align*}
\]


\[
\begin{align*}
F & \quad G & \quad [F] & \quad [G] & \quad F \land G & \quad F \lor G & \quad F & \quad F \Rightarrow G & \quad \forall x Fx & \quad \exists x Fx & \quad H & \quad H & \quad F & \quad \neg F & \quad F \\
F & \quad F & \quad G & \quad F & \quad H & \quad H & \quad F & \quad \neg F & \quad F & \quad \neg F & \quad F & \quad \neg F \\
\end{align*}
\]


One axiom \((F \lor \neg F)\) needed for obtaining completeness
RESOLUTION (Robinson 1965)

A negative test calculus

Uses formulas in form of clauses
(set of literals, implicitly disjoint)

• One axiom, elementary contradiction
  (empty clause)

• One resolution rule

Simplest form:  

| clause1:    | L,K₁,...,Kₙ |
| clause2:    | ¬L,M₁,...,Mₘ |
| resolvent:  | K₁,...,Kₙ,M₁,...,Mₘ |

Resolution with substitution:  

| clause1:    | L,K₁,...,Kₙ |
| clause2:    | ¬L',M₁,...,Mₘ |
| resolvent:  | σK₁,...,σKₙ,σM₁,...,σMₘ |

σL = σL'  

Resolution has refutation completeness
RESOLUTION - AN EXAMPLE

A barber shaves a person if and only if
that person does not shave himself.

*Formalization of the statement and normalization into disjunctive Normal form*

\[ \text{shave(barber,}x) \leftrightarrow \neg\text{shave}(x,x) \]

transformed into:

\[ (\text{shave(barber,}x) \rightarrow \neg\text{shave}(x,x)) \land (\neg\text{shave}(x,x) \rightarrow \text{shave(barber,}x)) \]

\[ (\neg\text{shave(barber,}x) \lor \neg\text{shave}(x,x)) \land (\text{shave}(x,x) \lor \text{shave(barber,}x)) \]

*Factorization prior to substitution:*

\( \sigma = \{x \leftarrow \text{barber}, y \leftarrow \text{barber} \} \)

\( \text{shave}(x,x), \text{shave(barber,}x) \vdash \text{shave(barber,}barber) \)

\( \neg\text{shave(barber,}y), \neg\text{shave(y,}y) \vdash \neg\text{shave(barber,}barber) \)
THEORY RESOLUTION

Extension of the resolution principle to a domain theory

No interpretation makes both $L$ and $\neg L$ true

• in resolution only for syntactic complementarity
• extended to (multiple) semantic complementarity

An example

clause1: $a < b$, $K$
clause2: $b < c$, $M$
clause3: $c < a$, $N$

resolvent: $K$, $M$, $N$
REPRESENTATION

Efficiency through reduction rules

• *logical simplifications*
  (tautologies replaced by true, contradictions by false, replacements similar to elimination rules in ND calculus)

• simplifications with *useless* clauses
  (e.g.: isolation rule – clause with 'unique' literal)

Design of reduction rules 'creative', needs justification

Consequences of representation

State transition instead of calculus

• *Initial* states are sets of clauses

• *Intermediate* states are e.g., clauses graphs

• *Terminal* states – with or without empty clause
CONTROL

Efficiency through syntactically-based strategies

• Restriction strategies
  (e.g., unit resolution – complete for Horn clauses, input resolution, set-of-support, …)

• Ordering strategies
  (e.g.: saturation level, with unit preference, fairness, unit resolution, …)

An example prover - OTTER

• Elaborate implementation, indexing techniques
• User categorises clauses into (ordinary) clauses, axioms, and set of support, specifies options
• Makes resolution with axioms and 'best' clause from set-of-support
DEDUCTIVE DATABASES

Components

- Tables (*existential* database)
- Rules (*intensional* database)

Rules define intensional database on the base of the extensional one

Procedure

- Elementary relations expressed in tables
- General, recursive relations expressed in rules
- Query evaluation through resolution derivation

Benefit

- Combines capabilities of relational databases and rule-based systems
- Strong reduction of redundancy and number of tables

Problem: Control over chained application of rules
REPRESENTATION IN A RELATIONAL DATABASE

Example database relation

<table>
<thead>
<tr>
<th>CNAME</th>
<th>PNAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith John Jr</td>
<td>Smith John</td>
</tr>
<tr>
<td>Smith John Jr</td>
<td>Smith Mary</td>
</tr>
<tr>
<td>Rogers Charles</td>
<td>Rogers Linda</td>
</tr>
<tr>
<td>Rogers Linda</td>
<td>Jones David</td>
</tr>
<tr>
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<td>Jones Mary</td>
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<td>Smith Mary</td>
<td>Ford Albert</td>
</tr>
<tr>
<td>Cramer Steven</td>
<td>Cramer William</td>
</tr>
</tbody>
</table>

Relational calculus query to find the parent(s) of Charles Rogers

- GET(X) : PARENT(Rogers Charles, X)
REPRESENTATION WITH TWO TABLES

Example database relation

<table>
<thead>
<tr>
<th>PARENT</th>
<th>GRANDPARENT</th>
</tr>
</thead>
<tbody>
<tr>
<td>CNAME</td>
<td>PNAME</td>
</tr>
<tr>
<td>Smith John Jr</td>
<td>Smith John</td>
</tr>
<tr>
<td>Smith John Jr</td>
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</tr>
<tr>
<td>Smith John Jr</td>
<td>Ford Albert</td>
</tr>
</tbody>
</table>

Relational calculus query to find the grandparent(s) of Charles Rogers

- GET(X) : GRANDPARENT(Rogers Charles, X)
REPRESENTATION IN A DEDUCTIVE DATABASE

Extensional database

<table>
<thead>
<tr>
<th>CNAME</th>
<th>PNAME</th>
</tr>
</thead>
<tbody>
<tr>
<td>Smith John Jr</td>
<td>Smith John</td>
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<td>Cramer Steven</td>
<td>Cramer William</td>
</tr>
</tbody>
</table>

Intensional database

∀X∀Y∀Z PARENT(X,Y) & PARENT(Y,Z) → GRANDPARENT(X,Z)

Relational calculus query to find the grandparent(s) of Charles Rogers

• GET(X) : GRANDPARENT(Rogers Charles, X)
REPRESENTATION AS LOGICAL FORMULAS

PARENT(Smith John Jr, Smith John)
PARENT(Smith John Jr, Smith Mary)
PARENT(Rogers Charles, Rogers Linda)
PARENT(Rogers Linda, Jones David)
PARENT(Rogers Linda, Jones Mary)
PARENT(Smith Mary, Ford Albert)
PARENT(Cramer Steven, Cramer William)

∀X∀Y∀Z PARENT(X,Y) & PARENT(Y,Z) → GRANDPARENT(X,Z)
PROOF OF ASSERTIONS WITH THE DATABASE

a) “Is Rogers Charles a parent of Rogers Linda?”
PROOF OF ASSERTIONS WITH THE DATABASE

a) “Is Rogers Charles a parent of Rogers Linda?”
\[ \neg \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
PROOF OF ASSERTIONS WITH THE DATABASE

a) “Is Rogers Charles a parent of Rogers Linda?”
   ¬PARENT(Rogers Charles, Rogers Linda)
   PARENT(Rogers Charles, Rogers Linda)
   []
PROOF OF ASSERTIONS WITH THE DATABASE

a) “Is Rogers Charles a parent of Rogers Linda?”
\[ \neg \text{PARENT}(\text{Rogers Charles, Rogers Linda}) \]

b) “Is Rogers Charles a grandparent of Jones Mary?”
PROOF OF ASSERTIONS WITH THE DATABASE

a) “Is Rogers Charles a parent of Rogers Linda?”
   \[\neg\text{PARENT}(\text{Rogers Charles, Rogers Linda})\]
   \[\neg\text{PARENT}(\text{Rogers Charles, Rogers Linda})\]
   \[
   \]
b) “Is Rogers Charles a grandparent of Jones Mary?”
   \[\neg\text{GRANDPARENT}(\text{Rogers Charles, Jones Mary})\]
PROOF OF ASSERTIONS WITH THE DATABASE

a) “Is Rogers Charles a parent of Rogers Linda?”
\[ \neg \text{PARENT}(\text{Rogers Charles, Rogers Linda}) \]
\[ \vdash \text{PARENT}(\text{Rogers Charles, Rogers Linda}) \]
\[ \square \]

b) “Is Rogers Charles a grandparent of Jones Mary?”
\[ \neg \text{GRANDPARENT}(\text{Rogers Charles, Jones Mary}) \]
\[ \neg \text{PARENT}(\text{X,Y}) \lor \neg \text{PARENT}(\text{Y,Z}) \lor \text{GRANDPARENT}(\text{X,Z}) \]
PROOF OF ASSERTIONS WITH THE DATABASE

a) “Is Rogers Charles a parent of Rogers Linda?”
   \[ \neg \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
   \[ \neg \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
   \[ [] \]

b) “Is Rogers Charles a grandparent of Jones Mary?”
   \[ \neg \text{GRANDPARENT}(\text{Rogers Charles}, \text{Jones Mary}) \]
   \[ \neg \text{PARENT}(X,Y) \lor \neg \text{PARENT}(Y,Z) \lor \text{GRANDPARENT}(X,Z) \]
   \[ \neg \text{PARENT}(\text{Rogers Charles},Y) \lor \neg \text{PARENT}(Y, \text{Jones Mary}) \]
PROOF OF ASSERTIONS WITH THE DATABASE

a) “Is Rogers Charles a parent of Rogers Linda?”
\[ \neg \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
\[ \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]

b) “Is Rogers Charles a grandparent of Jones Mary?”
\[ \neg \text{GRANDPARENT}(\text{Rogers Charles}, \text{Jones Mary}) \]
\[ \neg \text{PARENT}(X,Y) \lor \neg \text{PARENT}(Y,Z) \lor \text{GRANDPARENT}(X,Z) \]
\[ \neg \text{PARENT}(\text{Rogers Charles},Y) \lor \neg \text{PARENT}(Y, \text{Jones Mary}) \]
\[ \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
PROOF OF ASSERTIONS WITH THE DATABASE

a) “Is Rogers Charles a parent of Rogers Linda?”
\[\neg \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
\[\text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
\[\text{[]} \]

b) “Is Rogers Charles a grandparent of Jones Mary?”
\[\neg \text{GRANDPARENT}(\text{Rogers Charles}, \text{Jones Mary}) \]
\[\neg \text{PARENT}(X,Y) \lor \neg \text{PARENT}(Y,Z) \lor \text{GRANDPARENT}(X,Z) \]
\[\neg \text{PARENT}(\text{Rogers Charles},Y) \lor \neg \text{PARENT}(Y, \text{Jones Mary}) \]
\[\text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
\[\neg \text{PARENT}(\text{Rogers Linda}, \text{Jones Mary}) \]
PROOF OF ASSERTIONS WITH THE DATABASE

a) “Is Rogers Charles a parent of Rogers Linda?”
\[ \neg \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
\[ \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
\[ [] \]

b) “Is Rogers Charles a grandparent of Jones Mary?”
\[ \neg \text{GRANDPARENT}(\text{Rogers Charles}, \text{Jones Mary}) \]
\[ \neg \text{PARENT}(X,Y) \lor \neg \text{PARENT}(Y,Z) \lor \text{GRANDPARENT}(X,Z) \]
\[ \neg \text{PARENT}(\text{Rogers Charles},Y) \lor \neg \text{PARENT}(Y, \text{Jones Mary}) \]
\[ \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
\[ \neg \text{PARENT}(\text{Rogers Linda}, \text{Jones Mary}) \]
\[ \text{PARENT}(\text{Rogers Linda}, \text{Jones Mary}) \]
\[ [] \]
REPRESENTATION WITH TWO TABLES

Extensional database

<table>
<thead>
<tr>
<th>PARENT</th>
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<tbody>
<tr>
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Intensional database

∀x∀y PARENT(x,y) & MALE(y) → FATHER(x,y)
∀x∀y∀z PARENT(x,y) & PARENT(y,z) → GRANDPARENT(x,z)
∀x∀y∀z GRANDPARENT(x,y) & FATHER(z,y) → GRANDFATHER(x,y)
ANSWERING A QUERY WITH THE DATABASE (1)

“Who are the grandparents of Rogers Charles?”
ANSWERING A QUERY WITH THE DATABASE (1)

“Who are the grandparents of Rogers Charles?”

Finding instantiations for the variable in the query representation

¬GRANDPARENT(Rogers Charles, V)
ANSWERING A QUERY WITH THE DATABASE (1)

“Who are the grandparents of Rogers Charles?”

Finding instantiations for the variable in the query representation

\[ \neg \text{GRANDPARENT}(\text{Rogers Charles}, \text{V}) \]
\[ \neg \text{PARENT}(X,Y) \lor \neg \text{PARENT}(Y,Z) \lor \text{GRANDPARENT}(X,Z) \]
ANSWERING A QUERY WITH THE DATABASE (1)

“Who are the grandparents of Rogers Charles?”

Finding instantiations for the variable in the query representation

\[ \neg \text{GRANDPARENT}(\text{Rogers Charles}, V) \]

\[ \neg \text{PARENT}(X, Y) \lor \neg \text{PARENT}(Y, Z) \lor \text{GRANDPARENT}(X, Z) \]

\[ \neg \text{PARENT}(\text{Rogers Charles}, Y) \lor \neg \text{PARENT}(Y, V) \]
“Who are the grandparents of Rogers Charles?”

Finding instantiations for the variable in the query representation

¬GRANDPARENT(Rogers Charles, V)

¬PARENT(X,Y) ∨ ¬PARENT(Y,Z) ∨ GRANDPARENT(X,Z)

¬PARENT(Rogers Charles,Y) ∨ ¬PARENT(Y, V)

PARENT(Rogers Charles, Rogers Linda)
ANSWERING A QUERY WITH THE DATABASE (1)

“Who are the grandparents of Rogers Charles?”

Finding instantiations for the variable in the query representation

\[ \neg \text{GRANDPARENT}(\text{Rogers Charles}, V) \]

\[ \neg \text{PARENT}(X, Y) \lor \neg \text{PARENT}(Y, Z) \lor \text{GRANDPARENT}(X, Z) \]

\[ \neg \text{PARENT}(\text{Rogers Charles}, Y) \lor \neg \text{PARENT}(Y, V) \]

\[ \text{PARENT}(\text{Rogers Charles, Rogers Linda}) \]

\[ \neg \text{PARENT}(\text{Rogers Linda, V}) \]
ANSWERING A QUERY WITH THE DATABASE (1)

“Who are the grandparents of Rogers Charles?”

Finding instantiations for the variable in the query representation

\[ \neg \text{GRANDPARENT}(Rogers \ Charles, \ V) \]
\[ \quad \neg \text{PARENT}(X, Y) \lor \neg \text{PARENT}(Y, Z) \lor \text{GRANDPARENT}(X, Z) \]

\[ \neg \text{PARENT}(Rogers \ Charles, Y) \lor \neg \text{PARENT}(Y, V) \]
\[ \quad \text{PARENT}(Rogers \ Charles, Rogers \ Linda) \]

\[ \neg \text{PARENT}(Rogers \ Linda, V) \]
\[ \quad \text{PARENT}(Rogers \ Linda, Jones \ Mary) \]

[]
ANSWERING A QUERY WITH THE DATABASE (1)

“Who are the grandparents of Rogers Charles?”

Finding instantiations for the variable in the query representation

\[ \neg \text{GRANDPARENT}(\text{Rogers Charles}, V) \]
\[ \quad \neg \text{PARENT}(X,Y) \lor \neg \text{PARENT}(Y,Z) \lor \text{GRANDPARENT}(X,Z) \]
\[ \quad \neg \text{PARENT}(\text{Rogers Charles}, Y) \lor \neg \text{PARENT}(Y, V) \]
\[ \quad \quad \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
\[ \neg \text{PARENT}(\text{Rogers Linda}, V) \]
\[ \quad \text{PARENT}(\text{Rogers Linda}, \text{Jones Mary}) \]
\[ \quad \quad \text{PARENT}(\text{Rogers Linda}, \text{Jones David}) \]

[]  []
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”


ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

¬GRANDFATHER(Rogers Charles, X)
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

\[
\neg\text{GRANDFATHER}(\text{Rogers Charles}, \, X) \\
\neg\text{GRANDPARENT}(V, Y) \lor \neg\text{FATHER}(Z, Y) \lor \text{GRANDFATHER}(V, Y)
\]
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

\[ \neg\text{GRANDFATHER}(\text{Rogers Charles, X}) \]
\[ \quad \neg\text{GRANDPARENT}(V,Y) \lor \neg\text{FATHER}(Z,Y) \lor \text{GRANDFATHER}(V,Y) \]
\[ \neg\text{GRANDPARENT}(\text{Rogers Charles, X}) \lor \neg\text{FATHER}(Z,X) \]

Helmut Horacek
Inferencing in Artificial Intelligence and Computational Linguistics
SS 2014
Language Technology
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

¬GRANDFATHER(Rogers Charles, X)
\[\neg \text{GRANDPARENT}(V,Y) \lor \neg \text{FATHER}(Z,Y) \lor \text{GRANDFATHER}(V,Y)\]

¬GRANDPARENT(Rogers Charles, X) \lor \neg \text{FATHER}(Z, X)
\[\neg \text{PARENT}(V,Y) \lor \neg \text{PARENT}(Y,W) \lor \text{GRANDPARENT}(V,W)\]
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

¬GRANDFATHER(Rogers Charles, X)
   \[\neg\text{GRANDPARENT}(V,Y) \lor \neg\text{FATHER}(Z,Y) \lor \text{GRANDFATHER}(V, Y)\]

¬GRANDPARENT(Rogers Charles, X) \lor \neg\text{FATHER}(Z, X)
   \[\neg\text{PARENT}(V,Y) \lor \neg\text{PARENT}(Y,W) \lor \text{GRANDPARENT}(V, W)\]

¬PARENT(Rogers Charles, Y) \lor \neg\text{PARENT}(Y, X) \lor \neg\text{FATHER}(Z, X)
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

¬GRANDFATHER(Rogers Charles, X)

\[ \neg \text{GRANDPARENT}(V, Y) \lor \neg \text{FATHER}(Z, Y) \lor \text{GRANDFATHER}(V, Y) \]

¬GRANDPARENT(Rogers Charles, X) \lor \neg \text{FATHER}(Z, X)

\[ \neg \text{PARENT}(V, Y) \lor \neg \text{PARENT}(Y, W) \lor \text{GRANDPARENT}(V, W) \]

¬PARENT(Rogers Charles, Y) \lor \neg \text{PARENT}(Y, X) \lor \neg \text{FATHER}(Z, X)

\[ \neg \text{PARENT}(V, W) \lor \neg \text{MALE}(W) \lor \text{FATHER}(V, W) \]
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

\( \neg \text{GRANDFATHER} (\text{Rogers Charles, X}) \) \\
\( \neg \text{GRANDPARENT} (\text{V, Y}) \vee \neg \text{FATHER} (\text{Z, Y}) \vee \text{GRANDFATHER} (\text{V, Y}) \)

\( \neg \text{GRANDPARENT} (\text{Rogers Charles, X}) \vee \neg \text{FATHER} (\text{Z, X}) \) \\
\( \neg \text{PARENT} (\text{V, Y}) \vee \neg \text{PARENT} (\text{Y, W}) \vee \text{GRANDPARENT} (\text{V, W}) \)

\( \neg \text{PARENT} (\text{Rogers Charles, Y}) \vee \neg \text{PARENT} (\text{Y, X}) \vee \neg \text{FATHER} (\text{Z, X}) \) \\
\( \neg \text{PARENT} (\text{V, W}) \vee \neg \text{MALE} (\text{W}) \vee \text{FATHER} (\text{V, W}) \)

\( \neg \text{PARENT} (\text{Rogers Charles, Y}) \vee \neg \text{PARENT} (\text{Y, X}) \vee \neg \text{PARENT} (\text{Z, X}) \vee \neg \text{MALE} (\text{X}) \)
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

\[ \neg \text{GRANDFATHER}(\text{Rogers Charles}, X) \]
\[ \quad \quad \quad \quad \neg \text{GRANDPARENT}(V, Y) \lor \neg \text{FATHER}(Z, Y) \lor \text{GRANDFATHER}(V, Y) \]

\[ \neg \text{GRANDPARENT}(\text{Rogers Charles}, X) \lor \neg \text{FATHER}(Z, X) \]
\[ \quad \quad \quad \quad \neg \text{PARENT}(V, Y) \lor \neg \text{PARENT}(Y, W) \lor \text{GRANDPARENT}(V, W) \]

\[ \neg \text{PARENT}(\text{Rogers Charles}, Y) \lor \neg \text{PARENT}(Y, X) \lor \neg \text{FATHER}(Z, X) \]
\[ \quad \quad \quad \quad \neg \text{PARENT}(V, W) \lor \neg \text{MALE}(W) \lor \text{FATHER}(V, W) \]

\[ \neg \text{PARENT}(\text{Rogers Charles}, Y) \lor \neg \text{PARENT}(Y, X) \lor \neg \text{PARENT}(Z, X) \lor \neg \text{MALE}(X) \]

\[ \text{PARENT}(\text{Rogers Charles}, \text{Rogers Linda}) \]
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

\[ \neg \text{GRANDFATHER}(\text{Rogers Charles}, X) \]
\[ \quad \neg \text{GRANDPARENT}(V,Y) \lor \neg \text{FATHER}(Z,Y) \lor \text{GRANDFATHER}(V,Y) \]

\[ \neg \text{GRANDPARENT}(\text{Rogers Charles},X) \lor \neg \text{FATHER}(Z,X) \]
\[ \quad \neg \text{PARENT}(V,Y) \lor \neg \text{PARENT}(Y,W) \lor \text{GRANDPARENT}(V,W) \]

\[ \neg \text{PARENT}(\text{Rogers Charles},Y) \lor \neg \text{PARENT}(Y,X) \lor \neg \text{FATHER}(Z,X) \]
\[ \quad \neg \text{PARENT}(V,W) \lor \neg \text{MALE}(W) \lor \text{FATHER}(V,W) \]

\[ \neg \text{PARENT}(\text{Rogers Charles},Y) \lor \neg \text{PARENT}(Y,X) \lor \neg \text{PARENT}(Z,X) \lor \neg \text{MALE}(X) \]
\[ \quad \neg \text{PARENT}(\text{Rogers Charles, Rogers Linda}) \]
\[ \neg \text{PARENT}(\text{Rogers Linda, X}) \lor \neg \text{PARENT}(Z,X) \lor \neg \text{MALE}(X) \]
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

¬GRANDFATHER(Rogers Charles, X)

\[ ¬GRANDPARENT(V,Y) \lor ¬FATHER(Z,Y) \lor GRANDFATHER(V, Y) \]

¬GRANDPARENT(Rogers Charles, X) \lor ¬FATHER(Z, X)

\[ ¬PARENT(V,Y) \lor ¬PARENT(Y,W) \lor GRANDPARENT(V, W) \]

¬PARENT(Rogers Charles, Y) \lor ¬PARENT(Y, X) \lor ¬FATHER(Z, X)

\[ ¬PARENT(V,W) \lor ¬MALE(W) \lor FATHER(V, W) \]

¬PARENT(Rogers Charles, Y) \lor ¬PARENT(Y, X) \lor ¬PARENT(Z, X) \lor ¬MALE(X)

\[ PARENT(Rogers Charles, Rogers Linda) \]

¬PARENT(Rogers Linda, X) \lor ¬PARENT(Z, X) \lor ¬MALE(X)

\[ ¬PARENT(Rogers Linda, X) \lor ¬MALE(X) \]
ANSWERING A QUERY WITH THE DATABASE (2)

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\[\neg\text{PARENT}(V,Y) \lor \neg\text{PARENT}(Y,W) \lor \text{GRANDPARENT}(V, W)\]

¬PARENT(Rogers Charles, Y) \lor \neg\text{PARENT}(Y, X) \lor \neg\text{FATHER}(Z, X)

\[\neg\text{PARENT}(V,W) \lor \neg\text{MALE}(W) \lor \text{FATHER}(V, W)\]

¬PARENT(Rogers Charles, Y) \lor \neg\text{PARENT}(Y, X) \lor \neg\text{PARENT}(Z, X) \lor \neg\text{MALE}(X)

\[\text{PARENT}(\text{Rogers Charles, Rogers Linda})\]

¬PARENT(Rogers Linda, X) \lor \neg\text{PARENT}(Z, X) \lor \neg\text{MALE}(X)

¬PARENT(Rogers Linda, X) \lor \neg\text{MALE}(X)

\[\text{PARENT}(\text{Rogers Linda, Jones David})\]
ANSWERING A QUERY WITH THE DATABASE (2)

“Who is the grandfather of Rogers Charles?”

\[ \neg \text{GRANDFATHER}(\text{Rogers Charles, } X) \]
\[ \quad \quad \quad \neg \text{GRANDPARENT}(V, Y) \lor \neg \text{FATHER}(Z, Y) \lor \text{GRANDFATHER}(V, Y) \]

\[ \neg \text{GRANDPARENT}(\text{Rogers Charles, } X) \lor \neg \text{FATHER}(Z, X) \]
\[ \quad \quad \quad \neg \text{PARENT}(V, Y) \lor \neg \text{PARENT}(Y, W) \lor \text{GRANDPARENT}(V, W) \]

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\[ \quad \quad \quad \quad \text{PARENT}(\text{Rogers Linda, Jones David}) \]

\[ \neg \text{MALE}(\text{Jones David}) \]
ANSWERING A QUERY WITH THE DATABASE (2)

“How is the grandfather of Rogers Charles?”

¬GRANDFATHER(Rogers Charles, X)

\[\sim \text{GRANDPARENT}(V,Y) \lor \sim \text{FATHER}(Z,Y) \lor \text{GRANDFATHER}(V,Y)\]

¬GRANDPARENT(Rogers Charles, X) \lor \sim \text{FATHER}(Z, X)

\[\sim \text{PARENT}(V,Y) \lor \sim \text{PARENT}(Y,W) \lor \text{GRANDPARENT}(V,W)\]

¬PARENT(Rogers Charles, Y) \lor \sim \text{PARENT}(Y, X) \lor \sim \text{FATHER}(Z, X)

\[\sim \text{PARENT}(V,W) \lor \sim \text{MALE}(W) \lor \text{FATHER}(V,W)\]

¬PARENT(Rogers Charles, Y) \lor \sim \text{PARENT}(Y, X) \lor \sim \text{PARENT}(Z, X) \lor \sim \text{MALE}(X)

\[\text{PARENT}(\text{Rogers Charles, Rogers Linda})\]

¬PARENT(Rogers Linda, X) \lor \sim \text{PARENT}(Z, X) \lor \sim \text{MALE}(X)

\[\sim \text{PARENT}(\text{Rogers Linda, X}) \lor \sim \text{MALE}(X)\]

\[\text{PARENT}(\text{Rogers Linda, Jones David})\]

¬MALE(Jones David)

MALE(Jones David)
THE STATE OF DEDUCTIVE DATABASES

Problems with deductive databases

• No commercial deductive database (one company producing DDBMS had to close)
• Prolog implementations are much faster than deductive databases

Use of ideas of deductive databases

• Prolog is used successfully in industry
• Constraint logic programming is very successful in industry
• Ideas of deductive databases used in extensions to standard relational databases
• Answer set programming as a new logic programming formalism
A Proof of God

(Christoph Benzmüller and Bruno Woltzenlogel Paleo, 2013)
IN A NUTSHELL

Techniques

• Advanced proof techniques required
  (higher order, modal logic)
• Progress in use of logics obtained

Media interest

• Germany: Spiegel Online, FAZ, Die Welt, ...
• International: Austria, Italy, India, US, ...
HISTORICAL BACKGROUND

Ontological argument

- Deductive argument (for the existence of God)
- Starting from premises, justified by pure reasoning

Rich history of ontological arguments

- Pro: Descartes, Leibniz, Hegel, Gödel, …
- Against: Th. Aquinas, Kant, Frege, …

Gödels notion of god

"A God-like being possesses all positive properties"

->

"(Necessarily) God exists"

proved by Gödel on two hand-written pages
SCOTT'S VERSION OF GÖDEL'S AXIOMS, DEFINITIONS AND THEOREMS

A1 Either a property or its negation is positive, but not both: \[ \forall \phi [P(\neg \phi) \leftrightarrow \neg P(\phi)] \]

A2 A property necessarily implied by a positive property is positive: \[ \forall \phi \forall \psi [(P(\phi) \land \Box \forall x[\phi(x) \rightarrow \psi(x))] \rightarrow P(\psi)] \]

T1 Positive properties are possibly exemplified: \[ \forall \varphi [P(\varphi) \rightarrow \Diamond \exists x \varphi(x)] \]

D1 A God-like being possesses all positive properties: \[ G(x) \leftrightarrow \forall \phi [P(\phi) \rightarrow \phi(x)] \]

A3 The property of being God-like is positive: \[ P(G) \]

C Possibly, God exists: \[ \Diamond \exists x G(x) \]

A4 Positive properties are necessarily positive: \[ \forall \phi [P(\phi) \rightarrow \Box P(\phi)] \]

D2 An essence of an individual is a property possessed by it and necessarily implying any of its properties: \[ \phi \text{ ess. } x \leftrightarrow \phi(x) \land \forall \psi(\psi(x) \rightarrow \Box \forall y(\phi(y) \rightarrow \psi(y))) \]

T2 Being God-like is an essence of any God-like being: \[ \forall x [G(x) \rightarrow G \text{ ess. } x] \]

D3 Necessary existence of an individual is the necessary exemplification of all its essences: \[ NE(x) \leftrightarrow \forall \phi [\phi \text{ ess. } x \rightarrow \Box \exists y \phi(y)] \]

A5 Necessary existence is a positive property: \[ P(NE) \]

T3 Necessarily, God exists: \[ \Box \exists x G(x) \]
PROOF OVERVIEW
(IN NATURAL DEDUCTION STYLE)
PROOF DESIGN

State-of-the-art

- No prover for higher order modal logic exists
- Several (increasingly better and coordinated) provers for higher order logic exist (interactive and automated ones)

Overall strategy

- Embedding in higher order classical logic
  (based on experience with embedding first-order modal logic in higher order logic)
- Making use of higher-order logic theorem provers
- Interactive proof oriented on human-designed natural deduction proof

Assessment

A fully automated proof may be possible in about 3 years

Benzmüller
EMBEDDING IN HIGHER-ORDER LOGIC

QML \( \varphi, \psi \) ::= \ldots | \neg \varphi | \varphi \land \psi | \varphi \rightarrow \psi | \Box \varphi | \Diamond \varphi | \forall x \varphi | \exists x \varphi | \forall P \varphi

HOL

\( s, t \) ::= C | x | \lambda x s | st | \neg s | s \lor t | \forall x t

QML in HOL: QML formulas \( \varphi \) are mapped to HOL predicates \( \varphi_{s \rightarrow o} \)

\[\begin{align*}
\neg &= \lambda \varphi_{t \rightarrow 0} \lambda s_{t} \neg \varphi s \\
\land &= \lambda \varphi_{t \rightarrow 0} \lambda \psi_{t \rightarrow 0} \lambda s_{t} (\varphi s \land \psi s) \\
\rightarrow &= \lambda \varphi_{t \rightarrow 0} \lambda \psi_{t \rightarrow 0} \lambda s_{t} (\neg \varphi s \lor \psi s) \\
\Box &= \lambda \varphi_{t \rightarrow 0} \lambda s_{t} \forall u_{t} (\neg rsu \lor \varphi u) \\
\Diamond &= \lambda \varphi_{t \rightarrow 0} \lambda s_{t} \exists u_{t} (rsu \land \varphi u) \\
\forall &= \lambda h_{\mu \rightarrow (t \rightarrow 0)} \lambda s_{t} \forall d_{\mu} \text{ hds} \\
\exists &= \lambda h_{\mu \rightarrow (t \rightarrow 0)} \lambda s_{t} \exists d_{\mu} \text{ hds} \\
\forall &= \lambda H_{(\mu \rightarrow (t \rightarrow 0)) \rightarrow (t \rightarrow 0)} \lambda s_{t} \forall d_{\mu} \text{ Hds} \\
\text{valid} &= \lambda \varphi_{t \rightarrow 0} \forall w_{t} \varphi w
\end{align*}\]

The equations in \( Ax \) are given as axioms to the HOL provers!
(Remark: Note that we are here dealing with constant domain quantification)
# Course of the Proof

## Subproof

<table>
<thead>
<tr>
<th>Checking consistency</th>
<th>Nitpick (model checker)</th>
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<tbody>
<tr>
<td>Checking consistency Gödels original definition of D2</td>
<td>LEO II (ATP)</td>
</tr>
<tr>
<td>Proving T1 (positive properties ev. exemplified) is a theorem</td>
<td>LEO II (ATP)</td>
</tr>
<tr>
<td>Proving C (possibly, God exists) is a theorem</td>
<td>LEO II (ATP)</td>
</tr>
<tr>
<td>Proving T2 (being God-like is an essence …) is a theorem</td>
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</tr>
<tr>
<td>Proving T3 (necessarily God exists) is a theorem</td>
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<tr>
<td>Proving C2 (necessarily God exists) is a theorem</td>
<td>LEO II (ATP)</td>
</tr>
<tr>
<td>Checking axioms are consistent</td>
<td>Nitpick (model checker)</td>
</tr>
<tr>
<td>Checking Gödels original axioms are inconsistent</td>
<td>LEO II (ATP)</td>
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<td>Checking modal collaps</td>
<td>LEO II (ATP)</td>
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<td>Checking &quot;flawlessness of God&quot;</td>
<td>LEO II (ATP)</td>
</tr>
<tr>
<td>Proving Monotheism</td>
<td>TPS (ATP)</td>
</tr>
</tbody>
</table>
CRITICISM & OUTLOOK

Problematic assumptions

• Everything that is the case is so necessarily. ∀P.[P \rightarrow P]
  (follows from T2, T3, D2, proved by higher order ATPs)
  Then everything is determined, there is no free will …

• Either a property or its negation is positive
  in the morale sense, according to Gödel

Results

• Powerful infrastructure to reason in higher-order modal logic
• Several insights about the strength of logics needed or not needed
• Difficult benchmark problems for higher-order theorem provers
• Major step towards computer-assisted theoretical philosophy
• Further ontological arguments to be tested (in particular, related to Gödel)

(see http://page.mi.fu-berlin.de/cbenzmueller/, link presentations)
STATE OF AFFAIRS OF THEOREM PROVERS

Capabilities

• Occasional success with proofs of prominent theorems
  (usually tedious and extremely longish, but first known formal result)
• Some specialized provers
  (taxonomic reasoners, equation provers)
• Considerable progress in efficiency recently

Variety of uses

• Remote access to several ATPs (first-order, higher order)
• Calling several (distinct) provers in parallel (hoping at least one succeeds)
• Combining reasoning techniques (proving + computer algebra)
• Interactive proving (adding control for the prover, software verification)
• Proof planning – provers supported by proof schemas that encapsulate knowledge
ANSWERING INTENSIONAL QUESTIONS (1)

Alternative query formulations and answers

• *Which* managers at IBM earn more than 1,000,000 $?
  Smith, Miller, and Jones.

• *What* managers at IBM earn more than 1,000,000 $?
  Managers of rank A.

Building intensional answers

• Based on *deduction*
  Exploiting rule traces

• Based on *induction* (*inductive logic programming*)
  Reasoning over commonalities of extensional sets
ANSWERING INTENSIONAL QUESTIONS (2)

Motivation

- Checking consistency constraints (debugging a database)
- Producing informative descriptions rather than pure enumerations

Techniques for building intensional answers – idea

- Building a description of one item
- Successively adapting this description, such that
  - increasingly more items in the “solution set” are covered
  - no element outside the “solution set” is covered
  - performing simplifications/generalizations whenever appropriate

This task is the same as building referring expressions identifying sets of objects!
ANSWERING INTENSIONAL QUESTIONS – EXAMPLES

Checking consistency

“Which states have a capital?” – “All states.”
“Which cities have inhabitants?” – “All German cities.”
(Swiss cities on the border not modeled)

Producing informative descriptions

“Which cities are in a state that borders Austria?” – “All cities located in Bayern.”
“Which cities are bigger than München?” (Berlin, Hamburg)
“All cities located at highway A24 and which a river flows through.”

Limitations

Commonalities must be “total” (no disjunctions, no exceptions)
No concept indicating what is “of interest”
PARSING AS DEDUCTION

Connection between parsing and deduction

• Axiomatization of context-free grammars in definite clauses (subset of first-order logic)
• Identification of context-free parsing with deduction for a restricted class of definite clauses
• Extension to larger classes of definite clauses by replacing atomic grammar symbols by complex ones matched by unification – constraints specified by an argument
• Further extended to unification grammars

Parsing algorithms

• Offline – constraints after context-free parsing
• Online – constraints during context-free parsing (considered here)
DEFINITE CLAUSES

Definitions – definite clauses

\[ P \leftarrow Q_1 \land \ldots \land Q_n \]

- \( P \) is true if \( Q_1 \) and \( Q_n \) are true
- \( P \) is positive literal or head
- \( Q_1 \) … \( Q_n \) are negative literals or body
- Literals have the form \( p(t_1,\ldots,t_k) \) with predicate \( p \) (arity \( k \)) and \( t_i \) as arguments (terms)

Definitions – a program

- A set of clauses (input clauses) is a program
- A program defines the relations between the predicates appearing in the heads of the clauses
- A goal statement \( \leftarrow P \) requires finding provable instances of \( P \)
DEFINITE CLAUSES

Definite clause grammars

Context-free rule

\[ X \rightarrow \alpha_1 \ldots \alpha_n \]

translated to definite clause

\[ X(S_0,S_n) \leftarrow \alpha_1(S_0,S_1) \& \ldots \& \alpha_n(S_{n-1},S_n) \]

with variables \( S_i \) being string arguments (positions in the input string)

- Generalization by adding predicate arguments to string arguments

Deduction in definite clauses

Resolution

(1) \[ B \leftarrow A_1 \& \ldots \& A_m \]

(2) \[ C \leftarrow D_1 \& \ldots \& D_i \& \ldots \& D_n \]

if \( B \) and \( D_i \) are unifiable by substitution \( \sigma \), infer

(3) \[ \sigma[C \leftarrow D_1 \& \ldots \& D_{i-1} \& A_1 \& \ldots \& A_m \& \ldots \& D_{i+1} \& \ldots \& D_n] \]

Clause (3) is a derived clause, resolvent from (1) and (2)
**EARLEY DEDUCTION**

**Definitions**

- Definite clauses divided into *program* and *state*
- Program – set of *input* clauses, fixed
- State – set of *derived* clauses, nonunit clauses with one negative literal selected (initially the goal)

**Inference rules**

- *Instantiation* – selected literal of some clause unifies with a positive literal of a *nonunit* clause $C$ in the program, deriving the instantiation $\sigma[C]$
  ($\sigma$ is the most general unifier of the two literals)
- *Reduction* – selected literal of some clause unifies with a *unit* clause in the program or in the current state, deriving the clause $\sigma[C']$
  $C'$ is $C$ minus the selected literal ($\sigma$ is the most general unifier of the two literals)

**Techniques**

- Mixed top-down bottom-up mechanism
- Blockage of derived clauses subsumed by the state
- Handling gaps, dependencies by extra arguments
EXAMPLE DEDUCTION PROOF

Context free grammar

\[
\begin{align*}
S & \rightarrow \text{NP VP} & s(S_0,S) & \iff \text{np}(S_0,S_1) \& \text{vp}(S_1,S) \\
\text{NP} & \rightarrow \text{Det N} & \text{np}(S_0,S) & \iff \text{det}(S_0,S_1) \& \text{n}(S_1,S) \\
\text{Det} & \rightarrow \text{NP Gen} & \text{det}(S_0,S) & \iff \text{np}(S_0,S_1) \& \text{gen}(S_1,S) \\
\text{Det} & \rightarrow \text{Art} & \text{det}(S_0,S) & \iff \text{art}(S_0,S) \\
\text{Det} & \rightarrow \text{A} & \text{det}(S_0,S) & \iff \text{n}(S_0,S) \\
\text{VP} & \rightarrow \text{V NP} & \text{vp}(S_0,S) & \iff \text{v}(S_0,S_1) \& \text{n}(S_1,S)
\end{align*}
\]

Definite clause program

\[
(20) \quad \text{s}(S_0,S) \iff \text{np}(S_0,S_1) \& \text{vp}(S_1,S)
\]

\[
(21) \quad \text{np}(S_0,S) \iff \text{det}(S_0,S_1) \& \text{n}(S_1,S)
\]

\[
(22) \quad \text{det}(S_0,S) \iff \text{np}(S_0,S_1) \& \text{gen}(S_1,S)
\]

\[
(23) \quad \text{det}(S_0,S) \iff \text{art}(S_0,S)
\]

\[
(24) \quad \text{det}(S_0,S) \iff \text{n}(S_0,S)
\]

\[
(25) \quad \text{vp}(S_0,S) \iff \text{v}(S_0,S_1) \& \text{n}(S_1,S)
\]

Lexical categories

of the sentence

\[Agatha,'s husband, hit Ulrich\]

represented by the unit clauses

\[\text{n}(0,1) \& \text{gen}(1,2) \& \text{n}(2,3) \& \text{v}(3,4) \& \text{n}(4,5)\]

goal statement \[\text{ans} \iff \text{s}(0,5)\]

provable, if (26) is a sentence
EXAMPLE DEDUCTION PROOF (1)

\[ s(S_0, S) \iff \text{np}(S_0, S_1) \& \text{vp}(S_1, S) \quad (20) \]

\[ \text{np}(S_0, S) \iff \text{det}(S_0, S_1) \& \text{n}(S_1, S) \quad (21) \]

\[ \text{det}(S_0, S) \iff \text{np}(S_0, S_1) \& \text{gen}(S_1, S) \quad (22) \]

\[ \text{det}(S_0, S) \iff \text{art}(S_0, S) \quad (23) \]

\[ \text{det}(S_0, S) \quad (24) \]

\[ \text{vp}(S_0, S) \iff \text{v}(S_0, S_1) \& \text{np}(S_1, S) \quad (25) \]

\[ \text{ans} \iff s(0,5) \quad \text{goal statement} \quad (33) \]
EXAMPLE DEDUCTION PROOF (1)

\[
\begin{align*}
    s(S_0, S) & \iff np(S_0, S_1) \land vp(S_1, S) \quad (20) \\
    np(S_0, S) & \iff det(S_0, S_1) \land n(S_1, S) \quad (21) \\
    det(S_0, S) & \iff np(S_0, S_1) \land gen(S_1, S) \quad (22) \\
    det(S_0, S) & \iff art(S_0, S) \quad (23) \\
    det(S_0, S) & \iff v(S_0, S_1) \land np(S_1, S) \quad (24) \\
    ans & \iff s(0, 5) \quad \text{goal statement} \quad (33) \\
    s(0, 5) & \iff np(0, S_1) \land vp(S_1, 5) \quad (33) \text{ instantiates (20)} \quad (34)
\end{align*}
\]
EXAMPLE DEDUCTION PROOF (1)

\[
\begin{align*}
  s(S_0, S) & \iff np(S_0, S_1) \land vp(S_1, S) & (20) \\
  np(S_0, S) & \iff det(S_0, S_1) \land n(S_1, S) & (21) \\
  det(S_0, S) & \iff np(S_0, S_1) \land gen(S_1, S) & (22) \\
  det(S_0, S) & \iff art(S_0, S) & (23) \\
  vp(S_0, S) & \iff v(S_0, S_1) \land np(S_1, S) & (25) \\
  \text{ans} & \iff s(0, 5) & \text{goal statement} & (33) \\
  s(0, 5) & \iff np(0, S_1) \land vp(S_1, 5) & (33) \text{ instantiates } (20) & (34) \\
  np(0, S) & \iff det(0, S_1) \land n(S_1, S) & (34) \text{ instantiates } (21) & (35)
\end{align*}
\]
### Example Deduction Proof (1)

<table>
<thead>
<tr>
<th>Relation</th>
<th>Implication</th>
<th>Proof Line</th>
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</thead>
<tbody>
<tr>
<td>$s(S_0, S)$ &amp; $np(S_0, S_1) &amp; vp(S_1, S)$</td>
<td>$\Leftarrow$</td>
<td>(20)</td>
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<tr>
<td>$np(S_0, S)$ &amp; $det(S_0, S_1) &amp; n(S_1, S)$</td>
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<td>(21)</td>
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<tr>
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<td>(23)</td>
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<td>$\Leftarrow$</td>
<td>(24)</td>
</tr>
<tr>
<td>$ans$ &amp; $s(0,5)$</td>
<td>$\Leftarrow$</td>
<td>(33) &amp; goal statement</td>
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<tr>
<td>$s(0,5)$ &amp; $np(0, S_1) &amp; vp(S_1, 5)$</td>
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## Example Deduction Proof (1)

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<tbody>
<tr>
<td>( s(S_0, S) ) &amp; ( \text{np}(S_0, S) )</td>
<td>( \Leftrightarrow )</td>
<td>( \text{vp}(S_1, S) )</td>
</tr>
<tr>
<td>( \text{np}(S_0, S) ) &amp; ( \text{det}(S_0, S) )</td>
<td>( \Leftrightarrow )</td>
<td>( \text{n}(S_1, S) )</td>
</tr>
<tr>
<td>( \text{det}(S_0, S) ) &amp; ( \text{np}(S_0, S) )</td>
<td>( \Leftrightarrow )</td>
<td>( \text{gen}(S_1, S) )</td>
</tr>
<tr>
<td>( \text{det}(S_0, S) ) &amp; ( \text{np}(S_0, S) )</td>
<td>( \Leftrightarrow )</td>
<td>( \text{art}(S_0, S) )</td>
</tr>
<tr>
<td>( \text{vp}(S_0, S) ) &amp; ( \text{v}(S_0, S) )</td>
<td>( \Leftrightarrow )</td>
<td>( \text{vp}(S_1, S) )</td>
</tr>
<tr>
<td>( \text{ans} ) &amp; ( s(0, 5) )</td>
<td>( \Leftrightarrow )</td>
<td>( \text{goal statement} )</td>
</tr>
<tr>
<td>( s(0, 5) ) &amp; ( \text{np}(S_0, S) )</td>
<td>( \Leftrightarrow )</td>
<td>( \text{vp}(S_1, S) )</td>
</tr>
<tr>
<td>( \text{np}(0, S) ) &amp; ( \text{det}(S_0, S) )</td>
<td>( \Leftrightarrow )</td>
<td>( \text{n}(S_1, S) )</td>
</tr>
<tr>
<td>( \text{det}(0, S) ) &amp; ( \text{np}(S_0, S) )</td>
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</tr>
<tr>
<td>( \text{det}(0, S) ) &amp; ( \text{np}(S_0, S) )</td>
<td>( \Leftrightarrow )</td>
<td>( \text{art}(S_0, S) )</td>
</tr>
</tbody>
</table>
EXAMPLE DEDUCTION PROOF (1)

\[ s(S_0, S) \iff np(S_0, S_1) \land vp(S_1, S) \] \hspace{2cm} (20)

\[ np(S_0, S) \iff det(S_0, S_1) \land n(S_1, S) \] \hspace{2cm} (21)

\[ det(S_0, S) \iff np(S_0, S_1) \land gen(S_1, S) \] \hspace{2cm} (22)

\[ det(S_0, S) \iff art(S_0, S) \] \hspace{2cm} (23)

\[ vp(S_0, S) \iff v(S_0, S_1) \land np(S_1, S) \] \hspace{2cm} (24)

\[ ans \iff s(0, 5) \] \hspace{2cm} goal statement (33)

\[ s(0, 5) \iff np(0, S_1) \land vp(S_1, 5) \] \hspace{2cm} (33) instantiates (20) (34)

\[ np(0, S) \iff det(0, S_1) \land n(S_1, S) \] \hspace{2cm} (34) instantiates (21) (35)

\[ det(0, S) \iff np(0, S_1) \land gen(S_1, S) \] \hspace{2cm} (35) instantiates (22) (36)

\[ det(0, S) \iff art(0, S) \] \hspace{2cm} (35) instantiates (23) (37)

\[ np(0, S) \iff n(0, S) \] \hspace{2cm} (24) reduces (35) (38)
EXAMPLE DEDUCTION PROOF (1)

\[
\begin{align*}
s(S_0,S) & \iff np(S_0,S_1) & \land & vp(S_1,S) & (20) \\
np(S_0,S) & \iff det(S_0,S_1) & \land & n(S_1,S) & (21) \\
det(S_0,S) & \iff np(S_0,S_1) & \land & gen(S_1,S) & (22) \\
det(S_0,S) & \iff art(S_0,S) & (23) \\
vp(S_0,S) & \iff v(S_0,S_1) & \land & np(S_1,S) & (24) \\
ans & \iff s(0,5) & & & (33) \text{ goal statement} \\
s(0,5) & \iff np(0,S_1) & \land & vp(S_1,5) & (33) \text{ instantiates (20)} (34) \\
np(0,S) & \iff det(0,S_1) & \land & n(S_1,S) & (34) \text{ instantiates (21)} (35) \\
det(0,S) & \iff np(0,S_1) & \land & gen(S_1,S) & (35) \text{ instantiates (22)} (36) \\
det(0,S) & \iff art(0,S) & (35) \text{ instantiates (23)} (37) \\
np(0,S) & \iff n(0,S) & (24) \text{ reduces (35)} (38) \\
np(0,1) & \iff n(0,S) & (27) \text{ reduces (38)} (39)
\end{align*}
\]
### Example Deduction Proof (1)

<table>
<thead>
<tr>
<th>Rule</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>$s(S_0, S) \iff np(S_0, S_1) \land vp(S_1, S)$</td>
<td>(20)</td>
</tr>
<tr>
<td>$np(S_0, S) \iff det(S_0, S_1) \land n(S_1, S)$</td>
<td>(21)</td>
</tr>
<tr>
<td>$det(S_0, S) \iff np(S_0, S_1) \land gen(S_1, S)$</td>
<td>(22)</td>
</tr>
<tr>
<td>$det(S_0, S) \iff art(S_0, S)$</td>
<td>(23)</td>
</tr>
<tr>
<td>$vp(S_0, S) \iff v(S_0, S_1) \land np(S_1, S)$</td>
<td>(24)</td>
</tr>
<tr>
<td>$ans \iff s(0, 5)$</td>
<td>goal statement (33)</td>
</tr>
<tr>
<td>$s(0, 5) \iff np(0, S_1) \land vp(S_1, 5)$</td>
<td>(33) instantiates (20) (34)</td>
</tr>
<tr>
<td>$np(0, S) \iff det(0, S_1) \land n(S_1, S)$</td>
<td>(34) instantiates (21) (35)</td>
</tr>
<tr>
<td>$det(0, S) \iff np(0, S_1) \land gen(S_1, S)$</td>
<td>(35) instantiates (22) (36)</td>
</tr>
<tr>
<td>$det(0, S) \iff art(0, S)$</td>
<td>(35) instantiates (23) (37)</td>
</tr>
<tr>
<td>$np(0, S) \iff n(0, S)$</td>
<td>(24) reduces (35) (38)</td>
</tr>
<tr>
<td>$np(0, 1)$</td>
<td>(27) reduces (38) (39)</td>
</tr>
<tr>
<td>$s(0, 5) \iff vp(1, 5)$</td>
<td>(39) reduces (34) (40)</td>
</tr>
</tbody>
</table>
EXAMPLE DEDUCTION PROOF (1)

\[
s(S_0,S) \iff np(S_0,S_1) \& vp(S_1,S) \quad (20)
\]
\[
np(S_0,S) \iff det(S_0,S_1) \& n(S_1,S) \quad (21)
\]
\[
det(S_0,S) \iff np(S_0,S_1) \& gen(S_1,S) \quad (22)
\]
\[
det(S_0,S) \iff art(S_0,S) \quad (23)
\]
\[
vp(S_0,S) \iff v(S_0,S_1) \& np(S_1,S) \quad (24)
\]

\[
ans \iff s(0,5) \quad \text{goal statement} \quad (33)
\]
\[
s(0,5) \iff np(0,S_1) \& vp(S_1,5) \quad (33) \text{ instantiates (20)} \quad (34)
\]
\[
np(0,S) \iff det(0,S_1) \& n(S_1,S) \quad (34) \text{ instantiates (21)} \quad (35)
\]
\[
det(0,S) \iff np(0,S_1) \& gen(S_1,S) \quad (35) \text{ instantiates (22)} \quad (36)
\]
\[
det(0,S) \iff art(0,S) \quad (35) \text{ instantiates (23)} \quad (37)
\]
\[
np(0,S) \iff n(0,S) \quad (24) \text{ reduces (35)} \quad (38)
\]
\[
np(0,1) \quad (27) \text{ reduces (38)} \quad (39)
\]
\[
s(0,5) \iff vp(1,5) \quad (39) \text{ reduces (34)} \quad (40)
\]
\[
vp(1,5) \iff v(1,S_1) \& np(S_1,5) \quad (40) \text{ instantiates (25)} \quad (41)
\]
Example Deduction Proof (1)

\[
\begin{align*}
\text{s}(S_0, S) & \iff \text{np}(S_0, S_1) \land \text{vp}(S_1, S) \\
\text{np}(S_0, S) & \iff \text{det}(S_0, S_1) \land \text{n}(S_1, S) \\
\text{det}(S_0, S) & \iff \text{np}(S_0, S_1) \land \text{gen}(S_1, S) \\
\text{det}(S_0, S) & \iff \text{art}(S_0, S) \\
\text{vp}(S_0, S) & \iff \text{v}(S_0, S_1) \land \text{np}(S_1, S) \\
\text{ans} & \iff \text{s}(0, 5) \\
\text{s}(0, 5) & \iff \text{np}(0, S_1) \land \text{vp}(S_1, 5) \\
\text{np}(0, S) & \iff \text{det}(0, S_1) \land \text{n}(S_1, S) \\
\text{det}(0, S) & \iff \text{np}(0, S_1) \land \text{gen}(S_1, S) \\
\text{det}(0, S) & \iff \text{art}(0, S) \\
\text{np}(0, S) & \iff \text{n}(0, S) \\
\text{np}(0, 1) & \iff \text{vp}(1, 5) \\
\text{vp}(1, 5) & \iff \text{v}(1, S_1) \land \text{np}(S_1, 5) \\
\text{det}(0, S) & \iff \text{gen}(1, 5)
\end{align*}
\]

\begin{align*}
\text{(20)} & & \text{goal statement} \\
\text{(33)} & & \text{instantiates } (20) \\
\text{(34)} & & \text{instantiates } (21) \\
\text{(35)} & & \text{instantiates } (22) \\
\text{(36)} & & \text{instantiates } (23) \\
\text{(37)} & & \text{instantiates } (24) \\
\text{(38)} & & \text{reduces } (35) \\
\text{(39)} & & \text{reduces } (38) \\
\text{(40)} & & \text{reduces } (39) \\
\text{(41)} & & \text{instantiates } (25) \\
\text{(42)} & & \text{reduces } (36)
\end{align*}
EXAMPLE DEDUCTION PROOF (1)

\[

c(S_0, S) \iff np(S_0, S_1) \land vp(S_1, S) \\
np(S_0, S) \iff det(S_0, S_1) \land n(S_1, S) \\
det(S_0, S) \iff np(S_0, S_1) \land gen(S_1, S) \\
det(S_0, S) \iff art(S_0, S) \\
v(S_0, S) \iff v(S_0, S_1) \land np(S_1, S) \\

\]

\[
s(0, 5) \iff s(0, 5) \\
np(0, S) \iff np(0, S_1) \land vp(S_1, 5) \\
det(0, S) \iff det(0, S_1) \land n(S_1, S) \\
np(0, S) \iff np(0, S_1) \land gen(S_1, S) \\
det(0, S) \iff art(0, S) \\
np(0, S) \iff n(0, S) \\
np(0, 1) \iff n(0, S) \\
s(0, 5) \iff vp(1, 5) \\
v(1, S) \iff v(1, S_1) \land np(S_1, 5) \\
det(0, S) \iff gen(1, 5) \\
det(0, 2) \iff gen(1, 5)
\]

ans \iff s(0, 5) 
\]

\[
goal \; statement \\
(20) \; \text{instantiates (20)} \\
(21) \; \text{instantiates (21)} \\
(22) \; \text{instantiates (22)} \\
(23) \; \text{instantiates (23)} \\
(24) \; \text{instantiates (24)} \\
(25) \; \text{instantiates (25)} \\
(33) \; \text{instantiates (33)} \\
(34) \; \text{instantiates (34)} \\
(35) \; \text{instantiates (35)} \\
(36) \; \text{instantiates (36)} \\
(37) \; \text{instantiates (37)} \\
(38) \; \text{instantiates (38)} \\
(39) \; \text{instantiates (39)} \\
(40) \; \text{instantiates (40)} \\
(41) \; \text{instantiates (41)} \\
(42) \; \text{instantiates (42)} \\
(43) \; \text{instantiates (43)}
\]
EXAMPLE DEDUCTION PROOF (1)

\[
\begin{align*}
\text{s}(S_0,S) & \iff \text{np}(S_0,S_1) \& \text{vp}(S_1,S) & (20) \\
\text{np}(S_0,S) & \iff \text{det}(S_0,S_1) \& \text{n}(S_1,S) & (21) \\
\text{det}(S_0,S) & \iff \text{np}(S_0,S_1) \& \text{gen}(S_1,S) & (22) \\
\text{det}(S_0,S) & \iff \text{art}(S_0,S) & (23) \\
\text{vp}(S_0,S) & \iff \text{v}(S_0,S_1) \& \text{np}(S_1,S) & (24) \\
\text{ans} & \iff \text{s}(0,5) & \text{goal statement} & (33) \\
\text{s}(0,5) & \iff \text{np}(0,S_1) \& \text{vp}(S_1,5) & \text{(33) instantiates (20)} & (34) \\
\text{np}(0,S) & \iff \text{det}(0,S_1) \& \text{n}(S_1,S) & \text{(34) instantiates (21)} & (35) \\
\text{det}(0,S) & \iff \text{np}(0,S_1) \& \text{gen}(S_1,S) & \text{(35) instantiates (22)} & (36) \\
\text{det}(0,S) & \iff \text{art}(0,S) & \text{(35) instantiates (23)} & (37) \\
\text{np}(0,S) & \iff \text{n}(0,S) & \text{(24) reduces (35)} & (38) \\
\text{np}(0,1) & \iff \text{v}(1,S_1) \& \text{np}(S_1,5) & \text{(39) reduces (34)} & (40) \\
\text{s}(0,5) & \iff \text{vp}(1,5) & \text{(39) reduces (34)} & (40) \\
\text{vp}(1,5) & \iff \text{v}(1,S_1) \& \text{np}(S_1,5) & \text{(40) instantiates (25)} & (41) \\
\text{det}(0,S) & \iff \text{gen}(1,5) & \text{(39) reduces (36)} & (42) \\
\text{det}(0,2) & \iff \text{n}(2,5) & \text{(28) reduces (42)} & (43) \\
\text{np}(0,S) & \iff \text{n}(2,5) & \text{(43) reduces (35)} & (44)
\end{align*}
\]
EXAMPLE DEDUCTION PROOF (1)

\[
s(S_0, S) \iff np(S_0, S_1) & vp(S_1, S) \quad (20)
\]

\[
np(S_0, S) \iff det(S_0, S_1) & n(S_1, S) \quad (21)
\]

\[
det(S_0, S) \iff np(S_0, S_1) & gen(S_1, S) \quad (22)
\]

\[
det(S_0, S) \iff art(S_0, S) \quad (23)
\]

\[
vp(S_0, S) \iff v(S_0, S_1) & np(S_1, S) \quad (24)
\]

\[
ans \iff s(0,5) \quad \text{goal statement} \quad (33)
\]

\[
s(0,5) \iff np(0, S_1) & vp(S_1, 5) \quad (33) \text{ instantiates } (20) \quad (34)
\]

\[
np(0, S) \iff det(0, S_1) & n(S_1, S) \quad (34) \text{ instantiates } (21) \quad (35)
\]

\[
det(0, S) \iff np(0, S_1) & gen(S_1, S) \quad (35) \text{ instantiates } (22) \quad (36)
\]

\[
det(0, S) \iff art(0, S) \quad (35) \text{ instantiates } (23) \quad (37)
\]

\[
np(0, S) \iff n(0, S) \quad (24) \text{ reduces } (35) \quad (38)
\]

\[
np(0, 1) \quad (27) \text{ reduces } (38) \quad (39)
\]

\[
s(0,5) \iff vp(1,5) \quad (39) \text{ reduces } (34) \quad (40)
\]

\[
vp(1,5) \iff v(1, S_1) & np(S_1, 5) \quad (40) \text{ instantiates } (25) \quad (41)
\]

\[
det(0, S) \iff gen(1,5) \quad (39) \text{ reduces } (36) \quad (42)
\]

\[
det(0, 2) \quad (28) \text{ reduces } (42) \quad (43)
\]

\[
np(0, S) \iff n(2,5) \quad (43) \text{ reduces } (35) \quad (44)
\]

\[
np(0, 3) \quad (29) \text{ reduces } (44) \quad (45)
\]
EXAMPLE DEDUCTION PROOF (1)

\[
\begin{align*}
s(S0,S) & \iff \text{np}(S0,S1) \land \text{vp}(S1,S) & \quad (20) \\
\text{np}(S0,S) & \iff \text{det}(S0,S1) \land \text{n}(S1,S) & \quad (21) \\
\text{det}(S0,S) & \iff \text{np}(S0,S1) \land \text{gen}(S1,S) & \quad (22) \\
\text{det}(S0,S) & \iff \text{art}(S0,S) & \quad (23) \\
\text{vp}(S0,S) & \iff \text{v}(S0,S1) \land \text{np}(S1,S) & \quad (24) \\
\text{ans} & \iff \text{s}(0,5) & \quad \text{goal statement} \quad (33) \\
\text{s}(0,5) & \iff \text{np}(0,S1) \land \text{vp}(S1,5) & \quad (33) \text{ instantiates (20)} \quad (34) \\
\text{np}(0,S) & \iff \text{det}(0,S1) \land \text{n}(S1,S) & \quad (34) \text{ instantiates (21)} \quad (35) \\
\text{det}(0,S) & \iff \text{np}(0,S1) \land \text{gen}(S1,S) & \quad (35) \text{ instantiates (22)} \quad (36) \\
\text{det}(0,S) & \iff \text{art}(0,S) & \quad (35) \text{ instantiates (23)} \quad (37) \\
\text{np}(0,S) & \iff \text{n}(0,S) & \quad (24) \text{ reduces (35)} \quad (38) \\
\text{np}(0,1) & \iff \text{n}(0,S) & \quad (27) \text{ reduces (38)} \quad (39) \\
\text{s}(0,5) & \iff \text{vp}(1,5) & \quad (39) \text{ reduces (34)} \quad (40) \\
\text{vp}(1,5) & \iff \text{v}(1,S1) \land \text{np}(S1,5) & \quad (40) \text{ instantiates (25)} \quad (41) \\
\text{det}(0,S) & \iff \text{gen}(1,5) & \quad (39) \text{ reduces (36)} \quad (42) \\
\text{det}(0,2) & \iff \text{gen}(1,5) & \quad (28) \text{ reduces (42)} \quad (43) \\
\text{np}(0,S) & \iff \text{n}(2,5) & \quad (43) \text{ reduces (35)} \quad (44) \\
\text{np}(0,3) & \iff \text{n}(2,5) & \quad (29) \text{ reduces (44)} \quad (45) \\
\text{s}(0,5) & \iff \text{vp}(3,5) & \quad (45) \text{ reduces (34)} \quad (46)
\end{align*}
\]
**EXAMPLE DEDUCTION PROOF (2)**

\[
\begin{align*}
s(S_0,S) & \iff np(S_0,S_1) \land vp(S_1,S) \quad (20) \\
np(S_0,S) & \iff det(S_0,S_1) \land n(S_1,S) \quad (21) \\
det(S_0,S) & \iff np(S_0,S_1) \land gen(S_1,S) \quad (22) \\
det(S_0,S) & \iff art(S_0,S) \quad (23) \\
det(S_0,S) & \iff v(S_0,S_1) \land np(S_1,S) \quad (25) \\
det(0,S) & \iff gen(3,S) \quad (45) \text{ reduces } (36) \quad (47)
\end{align*}
\]
EXAMPLE DEDUCTION PROOF (2)

\[
\begin{align*}
    s(S_0, S) & \iff np(S_0, S_1) \& vp(S_1, S) \quad (20) \\
    np(S_0, S) & \iff det(S_0, S_1) \& n(S_1, S) \quad (21) \\
    det(S_0, S) & \iff np(S_0, S_1) \& gen(S_1, S) \quad (22) \\
    det(S_0, S) & \iff art(S_0, S) \quad (23) \\
    det(S_0, S) & \iff v(S_0, S_1) \& np(S_1, S) \quad (24) \\
    vp(S_0, S) & \iff v(S_0, S_1) \& np(S_1, S) \quad (25) \\
    det(0, S) & \iff gen(3, S) \quad (45) \text{ reduces } (36) \quad (47) \\
    vp(3, 5) & \iff v(3, S_1) \& np(S_1, 5) \quad (46) \text{ instantiates } (25) \quad (48)
\end{align*}
\]
## Example Deduction Proof (2)

\[
\begin{align*}
s(S_0,S) &\iff np(S_0,S_1) \land vp(S_1,S) & \text{(20)} \\
np(S_0,S) &\iff det(S_0,S_1) \land n(S_1,S) & \text{(21)} \\
det(S_0,S) &\iff np(S_0,S_1) \land gen(S_1,S) & \text{(22)} \\
det(S_0,S) &\iff art(S_0,S) & \text{(23)} \\
vp(S_0,S) &\iff v(S_0,S_1) \land np(S_1,S) & \text{(25)} \\
det(0,S) &\iff gen(3,S) & \text{(45) reduces (36)} \\
vp(3,5) &\iff v(3,S_1) \land np(S_1,5) & \text{(46) instantiates (25)} \\
vp(3,5) &\iff np(4,5) & \text{(30) reduces (48)}
\end{align*}
\]
EXAMPLE DEDUCTION PROOF (2)

\[
\begin{align*}
    s(S0,S) & \iff np(S0,S1) & \& vp(S1,S) \\
    np(S0,S) & \iff det(S0,S1) & \& n(S1,S) \\
    det(S0,S) & \iff np(S0,S1) & \& gen(S1,S) \\
    det(S0,S) & \iff art(S0,S) \\
    det(S0,S) & \iff v(S0,S1) & \& np(S1,S) \\
    \end{align*}
\]

\[
\begin{align*}
    det(0,S) & \iff gen(3,S) & \quad (45) \text{ reduces } (36) \\
    vp(3,5) & \iff v(3,S1) & \& np(S1,5) & \quad (46) \text{ instantiates } (25) \\
    vp(3,5) & \iff np(4,5) & \quad (30) \text{ reduces } (48) \\
    np(4,5) & \iff det(4,S1) & \& n(S1,5) & \quad (49) \text{ instantiates } (21) \\
\end{align*}
\]
**Example Deduction Proof (2)**

<table>
<thead>
<tr>
<th>Rule</th>
<th>Implication</th>
</tr>
</thead>
<tbody>
<tr>
<td>(s(S_0, S)) &amp; (\text{np}(S_0, S_1) &amp; \text{vp}(S_1, S)) (\iff)</td>
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\end{align*}
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<th>Equivalence</th>
<th>Proof Step</th>
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v(S_0,S) & \iff v(S_0,S_1) \land np(S_1,S) \quad (24) \\
\end{align*}
\]

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det(0,S) & \iff gen(3,S) \quad (45) \text{ reduces } (36) \quad (47) \\
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EXAMPLE DEDUCTION PROOF (2)

s(S0,S) ⇐ np(S0,S1) & vp(S1,S) \hspace{1cm} (20)
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\text{ans} & \iff \quad (59) \text{ reduces } (33) \quad (60)
\end{align*}
\]
PRESENTING DEDUCTIVE REASONING

Candidates for reasoning steps

• Resolution unintuitive
• Natural deduction too detailed systematic transformations between them
• Assertion level as application of axioms

Modifications

• Some sorts of logical consequences preferably conveyed implicitly through discourse context and default expectations (e.g., 'direct' instantiations)

• Divergent human performance in comprehending deductive syllogisms
  – 91% correct conclusions for modus ponens
  – 64% for modus tollens,
  – 48% for affirmative disjunction,
  – 30% for negative disjunction

• Logically redundant pieces of information reintroduced to support the addressee's attention in hard inferences

Direct proof verbalizations communicatively inadequate!
**EXAMPLE 1**

*Verbose and communicatively redundant text*

1. “Let \( \rho \) be an equivalence relation.
   Therefore we have \( \rho \) is reflexive, we have \( \rho \) is symmetric, and we have \( \rho \) is transitive.
   Then we have \( \rho \) is symmetric and we have \( \rho \) is reflexive.
   Then \( \forall x: x \rho x \).
   Thus we have \( h_0^y \rho h_0^y \). ... “
**EXAMPLE 1**

*Verbose and communicatively redundant text*

1. “Let $\rho$ be an equivalence relation. Therefore we have $\rho$ is reflexive, we have $\rho$ is symmetric, and we have $\rho$ is transitive. Then we have $\rho$ is symmetric and we have $\rho$ is reflexive. Then $\forall x: x \rho x$. Thus we have $h_0 y_0 \rho h_0 y_0 \ldots$ “

*Exploiting*

- domain knowledge (about equivalence relations)
- inference capabilities (and-eliminations)

*Concise and communicatively adequate text*

1'. “Let $\rho$ be an equivalence relation. Thus we have $h_0 y_0 \rho h_0 y_0 \ldots$ “
EXAMPLE 2

Verbose, communicatively redundant, poorly focused text

(2) “Let $1 < a$. Since lemma 1.10 holds, $0 < a^{-1}$. Since $1 < a$ holds and '$<$' is monotone, $1a^{-1} < aa^{-1}$ holds. $a^{-1} < aa^{-1}$ because of the unit element of $K$. $aa^{-1} = 1$ because of the inverse element of $K$ for $a \neq 0$. Thus $a^{-1} < 1$. “
EXAMPLE 2

Verbose, communicatively redundant, poorly focused text

(2) “Let $1 < a$. Since lemma 1.10 holds, $0 < a^{-1}$.

Since $1 < a$ holds and '$<$' is monotone, $1a^{-1} < aa^{-1}$ holds.

$a^{-1} < aa^{-1}$ because of the unit element of $K$.

$aa^{-1} = 1$ because of the inverse element of $K$ for $a \neq 0$.

Thus $a^{-1} < 1$.”

Exploiting

• domain knowledge (group axioms)
• inference capabilities (their place of application)

Concise and communicatively adequate text

(2') “Let $1 < a$. Since lemma 1.10 holds, $0 < a^{-1}$.

Then $0 < a^{-1} = 1a^{-1} < aa^{-1} = 1$.”
Example 3

*Hardly comprehensible, communicatively inadequate text*

(3) “Let ρ be a transitive relation and let ¬ (a ρ b).
Let us assume that c ρ b.
Hence we have ¬ (a ρ c).“
EXAMPLE 3

Hardly comprehensible, communicatively inadequate text

(3) “Let $\rho$ be a transitive relation and let $\neg (a \rho b)$. Let us assume that $c \rho b$. Hence we have $\neg (a \rho c)$.”

Taking into account

• human memory limitations
  (performing and chaining inferences)

• degrees of involvedness of an inference
  (modus tollens + or-elimination)

Sufficiently explicit and communicatively adequate text

(3’) “Let $\rho$ be a transitive relation and let $\neg (a \rho b)$. Let us assume that $c \rho b$. Since $\rho$ is transitive, $\neg (a \rho b)$ implies that $\neg (a \rho c)$ or $\neg (c \rho b)$ holds. Since we have $\neg (a \rho b)$ and $c \rho b$, $\neg (a \rho c)$ follows.”
A PROBLEM – INTERLEAVING SUBSTRUCTURES

Structural discrepancies (lifting non-elementary text spans)

Too many players hit an acceptable shot,
then stand around admiring it and
wind up losing the point.
There is no time in an action game like tennis to applaud yourself and
still get in position for the next shot.
And you always have to assume there will be a next shot.

[Mann, Thompson 1987b]

Challenges

- Inferring the precise scope of arguments – contrasts projection (in analysis)
- Restructuring argumentative structures preserving sequences (in generation)
INTERLEAVING SUBSTRUCTURES – EXAMPLE (1)

The semantically "precise" variant

Too many players hit an acceptable shot and wind up losing the point. There is no time in an action game like tennis to applaud yourself and still get in position for the next shot. And you always have to assume there will be a next shot.

Then stand around admiring it.

SEQUENCE

ANTITHESIS

ELABORATION

N

N

N

N

S

S

N

N

N

Helmut Horacek
Inferencing in Artificial Intelligence and Computational Linguistics

SS 2014
Language Technology
The rhetorically adequate variant

Too many players hit an acceptable shot then stand around admiring it and wind up losing the point.

And you always have to assume there will be a next shot.

There is no time in an action game like tennis to applaud yourself and still get in position for the next shot.
EVIDENCE FOR BUILDING EXPLANATIONS

*Significant discrepancies between proof content and explanation content*

- Some proof portions perceived as redundant
- Some proof steps perceived as involved

*Issues to be addressed*

- Examine discrepancies and their reasons
- Develop techniques/procedures that bridge these discrepancies
**REPRESENTATION CONCEPTS**

**Machine-oriented representations**
- Uniformity
- Explicitness
- Technical reference

**Human-oriented presentations**
- Diversification
- Selectivity
- Conceptual/linguistic references

**Examples**
- Chess endgame databases – regularities not captured, hardly understood
- Expert system presentations

Commonality-exploiting and rhetorically-oriented recasting

Significant performance improvement in a tutoring system [Di Eugenio et al. 2005]
EXPLICITNESS VERSUS SELECTIVITY

Some sorts of logical consequences preferably conveyed implicitly

through discourse context and default expectations (e.g., 'direct' instantiations)

[Thüring, Wender 1985]

Modus ponens communicated as *Modus brevis*

[Sadock 1977], [Cohen 1987]

Some kinds of "easy" inferable consequences

• Taxonomic inferences (category memberships)

• Normal consequences of actions

• Contextually suitable instantiation of rules/regularities mentioned

• Responsible causes if sufficiently salient
USE IN TUTORING ENVIRONMENTS

Building “master” proofs

Deductive reasoning normally done as resolution-based proofs

Transformations to human-oriented representations prior to presentation

Methods for complete proofs inadequate for incremental proof construction
(machine-found proofs may be unintuitive, incrementality, proof step checking)

Semi-automated proof planning

Proof alternatives built off-line on a human-oriented representation level

Incremental proof construction

Matching partial proof step specifications with prefabricated representations

Identification of referred proofs steps resp. subproofs (may be ambiguous)

Verification yields completion of specifications (including direction of reasoning)
OBSTACLES TO EXPLANATIONS

Emphasis in problem-solving on efficiency

Aiming at refutation rather than construction

Disregarding reasoning paths not contributing to a solution

Some reasoning mechanisms widely “explanation-resistent”

(e.g., constraints, neural nets)

Combination of methods

Problem-solving with heterogeneous methods

Dependency on request types (pure problem-solving versus beyond)