BSCA-F: Efficient Fuzzy Valued Stable Coalition Forming Among Agents

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Abstract

We propose a new low-complexity coalition forming algorithm, BSCA-F, that enables agents to negotiate bilateral Shapley value stable coalitions in uncertain environments, and demonstrate its usefulness by example. In particular, we show that utilizing the possibilistic mean value for defuzzifying negotiated fuzzy agent payoffs is reasonable, and fuzzy ranking methods can be utilized to implement optimistic, or pessimistic strategies of individual agents.

1 Introduction

Game-theoretic coalition algorithms can be used by intelligent agents as coordination means in a variety of applications in different environments. Classic approaches such as in [12, 8] proposed solutions to the problem of how self-interested agents best form stable coalitions in the sense of cooperative game theory. However, negotiation during the coalition-forming process might be uncertain. Such uncertainties could be caused by the possibility of nondeterministic events that hamper the negotiation process and produce incomplete information. Agents might have uncertain knowledge about the share of coalition income in which they intend to participate or on the degree of their membership in one or multiple coalitions. An agent might determine the degree of its membership to potential coalitions by individually leveled commitments to other agents or bargains that indicate the degree of collaboration that the agents desire. The first case might imply the formation of fuzzy-valued coalitions, whereas the second case might induce the formation of fuzzy coalitions, which might partially overlap. In this paper, we focus on negotiations of game-theoretically stable fuzzy-valued coalitions.

In [5], agents learn about each other in a way that allows for uncertain environmental knowledge and different expectations of coalition values by different agents. However, coalition stability is based on the exponential Bayesian Core which may be empty in certain cases. [9] present a heuristic approach that avoids computing stable payoffs in the sense of cooperative game theory at all. Based on the work of [10], in [1] a possibilistic fuzzy extension of the Kernel is defined that allows agents to negotiate fuzzy Kernel stable coalitions with low polynomial complexity. However, the reduction in computational complexity strictly depends on the maximum size of coalitions allowed.

In this paper, we propose an alternative algorithm, BSCA-F, that allow agents to negotiate fuzzy-valued coalitions in the setting of possibility theory. The BSCA-F does not require to constrain coalition sizes for negotiations of low computational and communication complexity. To achieve this, we utilize the fuzzy bilateral Shapley value which, however, implies that, in general, only subgame-stability can be achieved. We show that it is reasonable to utilize the possibilistic mean value [3] for defuzzifying the negotiated fuzzy payoffs to implement unambiguous coalition contracts among the agents.

The remainder of this paper is structured as follows.

2 Background

We extend game-theoretic concepts of coalition theory by means of possibilistic interpretation of fuzzy coalition values [10], and fuzzy ranking methods. Possibility theory [13, 7] evolved from a possibilistic interpretation of fuzzy set theory. There is empirical evidence for that people perform rather possibilistic than probablistic reasoning though their subjective assessment of the probability and possibility of real world events closely coincide [11]. Possibilistic interpretation of any fuzzy quantity indicates the degree of its possibility but not the probabilistic degree of its truth. As a consequence, by modeling uncertainty in terms of fuzzy quantities negotiating agents may ignore certain situations they either do not understand, or are simply not interested in, rather than being enforced to assign individual probability values to each of them.

2.1 Fuzzy Quantities

In the following, we define fuzzy quantities, operations, and ranking methods that are needed to understand the notion fuzzy-valued coalition game we introduce in subsequent section. For more details, we refer the reader to, for example, [10, 1].

Definition 2.1 *A* fuzzy subset $\widetilde{s}$ *of a set* S *is defined by its* membership function $\mu_{\widetilde{s}} : S \mapsto [0, 1]$ *where* $\mu_{\widetilde{s}}(x), x \in S$ *is called the* degree of membership *or* membership value *of* x *in* S. $x \in \widetilde{s}$ *iff* $\mu_{\widetilde{s}}(x) > 0$, *with* $support(\widetilde{s}) := \{x \mid x \in \widetilde{s}\}$. $\mu_{\widetilde{s}}(x)$ *is also called* possibility distribution for X *(*$\Pi(X = x)$*), if it denotes the degree of possibility that variable* X *of domain* S *takes the value* x. *Any fuzzy subset of* $\mathbb{R}$ *is called a* fuzzy quantity.

Definition 2.2 *Let* $\widetilde{x}$ *a fuzzy quantity;* $\widetilde{\mathbb{R}}$ *the set of all fuzzy quantities.*

1. $size(\widetilde{x}) := \sup\{support(\widetilde{x})\} - \inf\{support(\widetilde{x})\}$
2. $\widetilde{x}$ *is* normalized *iff* $\sup_{x\in\mathbb{R}}\{\mu_{\widetilde{x}}(x)\} = 1$
3. $x \in \mathbb{R}$ *with* $\mu_{\widetilde{x}}(x) = \max_{y\in\mathbb{R}}(\mu_{\widetilde{x}}(y))$ *is a* modal value *of* $\widetilde{x}$.
4. $\widetilde{r}, r \in \mathbb{R}$ *denotes a fuzzy quantity with*

$$\mu_{\widetilde{r}}(x) = \begin{cases} 1 & \text{if } x = r \\ 0 & \text{otherwise} \end{cases}, \ x \in \mathbb{R}$$

5. *A* fuzzy interval $\widetilde{I}$ *is a fuzzy quantity with* $\forall x_1, x_2, x_3 \in \mathbb{R}, x_1 < x_2 < x_3 \ : \ \mu_{\widetilde{I}}(x_2) \geq \min(\mu_{\widetilde{I}}(x_1), \mu_{\widetilde{I}}(x_3))$
6. *A* trapezoid fuzzy interval $((x_1, \widehat{x_2, x_3}, x_4))$, x_1, x_2, x_3, $x_4 \in \mathbb{R}$ *is a fuzzy interval with* $\forall r \in \mathbb{R}$*:*

$$\mu_{((x_1,\widehat{x_2,x_3},x_4))}(r) = \begin{cases} 1 & \text{if } x_2 \leq r \leq x_3 \\ \frac{r-x_1}{x_2-x_1} & \text{if } x_1 < r < x_2 \\ \frac{x_4-r}{x_4-x_3} & \text{if } x_3 < r < x_4 \\ 0 & \text{otherwise} \end{cases}$$

Arithmetic operations on fuzzy quantities follow Zadeh's *extension principle*.

Definition 2.3 *Let* $\widetilde{x} \in \widetilde{\mathbb{R}}^n, n \in \mathbb{N}$. *The function* $\widetilde{f} : \widetilde{\mathbb{R}}^n \mapsto \widetilde{\mathbb{R}}$ *is called a* fuzzy extension *of a function* $f : \mathbb{R}^n \mapsto \mathbb{R}$ *iff* $\forall x \in \mathbb{R}^n : \mu_{\widetilde{f}(\widetilde{x})}(x) = \sup_{y\in\mathbb{R}^n}\{\min_{1\leq i\leq n}\{\mu_{\widetilde{x}_i}(y_i)\} \mid f(y) = x)\}$ *if* $f^{-1}(x) \neq \emptyset$, *and* $\mu_{\widetilde{f}(\widetilde{x})}(x) = 0$ *if* $f^{-1}(x) = \emptyset$.

Definition 2.4 *Let* $F_1, F_2 \in \mathbb{R}^F, \ x, y, z, a \in \mathbb{R}$. *Applying the extension principle, we define*

$$\begin{aligned} \mu_{F_1\oplus F_2}(x) &:= \sup\{\min(\mu_{F_1}(y), \mu_{F_2}(z)) \mid y + z = x\} \\ \mu_{-F_1}(x) &:= \mu_{F_1}(-x) \\ \mu_{F_1\ominus F_2}(x) &:= \mu_{F_1\oplus(-F_2)}(x) \\ \mu_{a\cdot F_1}(x) &:= \begin{cases} \mu_{F_1}(x/a) & \text{if } a \neq 0 \\ \mu_{\widetilde{0}} & \text{if } a = 0 \end{cases} \end{aligned}$$

Agents are supposed to negotiate coalitions with expected fuzzy gains, hence to compute, compare, and select fuzzy utility values. That, in particular, requires means of ranking fuzzy quantities several of which being proposed for different applications such as in [2]. We adopt fuzzy quantity ranking operators that have been introduced by Dubois and Prade in the setting of possibility theory [6].

Definition 2.5 *Let* $F_1, F_2 \in \mathbb{R}^F$, R *a fuzzy subset of* $\widetilde{\mathbb{R}} \times \widetilde{\mathbb{R}}$. R *is a* fuzzy ranking operator, *or* fuzzy similarity relation, *if* $\mu_R(F_1, F_2)$ *denotes the degree to which* F_1 *is "greater" or "similar" than* F_2*, respectively. Further, let* G *a fuzzy ranking operator and* S *a fuzzy similarity relation. We define* $(F_1 \widetilde{\geq}_G F_2) := \mu_G(F_1, F_2)$ *and* $(F_1 \widetilde{=}_S F_2) := \mu_S(F_1, F_2)$. *Regarding the possibility distributions* $F_1, F_2 \in \widetilde{\mathbb{R}}$ *of* f_1 *and* f_2*, respectively, [6] define the*

1. possibility of dominance $\widetilde{\geq}_P$ *of* f_1 *over* f_2 *as* $\Pi(f_1 \geq f_2) = F_1 \widetilde{\geq}_P F_2 = \sup\{\min(\mu_{F_1}(x), \mu_{F_2}(y)) \mid x, y \in \mathbb{R}, x \geq y\}$*;*
2. necessity of dominance $\widetilde{\geq}_N$ *of* f_1 *over* f_2 *as* $N(f_1 \geq f_2) = F_1 \widetilde{\geq}_N F_2 = \inf_x\{\sup_y\{\max(1 - \mu_{F_1}(x), \mu_{F_2}(y)) \mid x, y \in \mathbb{R}, x \geq y\}\}$*;*
3. possibility of strict dominance $\widetilde{>}_P$ *of* f_1 *over* f_2 *as* $\Pi(f_1 > f_2) = F_1 >_P F_2 = \sup_x\{\inf_y\{\min(\mu_{F_1}(x), 1 - \mu_{F_2}(y)) \mid x, y \in \mathbb{R}, x \leq y\}\}$*;*
4. necessity of strict dominance $\widetilde{>}_N$ *of* f_1 *over* f_2 *as* $N(f_1 > f_2) = F_1 >_N F_2 = \inf\{\max(1 - \mu_{F_1}(x), 1 - \mu_{F_2}(y)) \mid x, y \in \mathbb{R}, x \leq y\}$*;*
5. possibility of equality $\widetilde{=}_P$ *of* f_1 *and* f_2 *as* $\Pi(f_1 = f_2) = F_1 \widetilde{=}_P F_2 = \min((F_1 \widetilde{\geq}_P F_2), (F_2 \widetilde{\geq}_P F_1))$*;*
6. necessity of equality $\widetilde{=}_N$ *of* f_1 *and* f_2 *as* $N(f_1 = f_2) = F_1 \widetilde{=}_N F_2 = \min(\min(N(F_2 \geq F_1), 1 - \Pi(F2 > F_1)), \min(N(F_1 \geq F_2), 1 - \Pi(F1 > F_2)))$

A fuzzy set of maximal elements of a set X of fuzzy quantities X, and fuzzy logical operators "AND" and "OR" with operands in $[0, 1]$ are defined as follows.

Definition 2.6 *Let* X *a set of fuzzy quantities,* G *a fuzzy ranking operator,* $x, y \in [0, 1]$, $n \in \mathbb{N}$.

1. *The fuzzy subset* $\widetilde{\max}^G X$ *of* X *is given by* $\mu_{\widetilde{\max}^G X}(F_1) := \min_{F_2 \in X, F_2 \neq F_1}(F_1 \widetilde{\geq}_G F_2), F_1 \in X$ *denoting the degree to which* F_1 *is a maximal element of* X.
2. *The (crisp) set* $\max^G X$ *of maximal elements of* X *is defined as* $\max^G X := \{F \mid \mu_{\max^G X}(F) = \max \mu_{\max^G X}\}$
3. $x \widetilde{\bigwedge} y := \min(x, y)$, $x \widetilde{\bigvee} y := \max(x, y)$. *For* $\widetilde{\odot} \in \{\widetilde{\bigwedge}, \widetilde{\bigvee}\}$*:* $\widetilde{\odot}\{\widetilde{x}_1, \ldots, \widetilde{x}_n\} := (\widetilde{x}\widetilde{\odot}(\widetilde{x}_2 \ldots (\widetilde{x}_{n-1}\widetilde{\odot}\widetilde{x}_n)\ldots))$

Definition 2.7 *For* $\widetilde{x} \in \widetilde{\mathbb{R}}$, $L_\alpha(\widetilde{x}) := \{x \mid x \in \mathbb{R}, \mu_F(x) \geq \alpha\}$ *with* $\alpha \in [0, 1]$ *is called an* α*-level cut of* $\widetilde{x}$*. We also define* $L_\alpha(\widetilde{x}_* := \inf\{L_\alpha(\widetilde{x})\}$ *and* $L_\alpha(\widetilde{x}^* := \sup\{L_\alpha(\widetilde{x})\}$

Definition 2.8 *Given a fuzzy interval* $\widetilde{x} \in \widetilde{\mathbb{R}}$*,with* $\mu_{\widetilde{X}}$ *representing a possibility distribution for a variable* $X \in \mathbb{R}$,

$$E(X) := \int_0^1 \alpha(L_\alpha(\widetilde{x})_* + L_\alpha(\widetilde{x})^*)d\alpha$$

is called the possibilistic mean value *of X [3]. Instead of* $E(X)$*, we also write* $e(\widetilde{x})$.

The additive possibilistic mean value e is similar to the expected value of stochastic variables used in probability theory, though there is no common agreement on the semantics of exact degrees of possibility [1]. Since e maps fuzzy membership functions to crisp real values, we consider it as an appropriate method for defuzzification of fuzzy quantities in the setting of possibility theory.

Remark 2.9 *For any trapezoid fuzzy interval* $\widetilde{I} = (\widehat{(x_1, x_2, x_3, x_4)})$*,* $x_1, x_2, x_3, x_4 \in \mathbb{R}$,

$$e(\widetilde{I}) = \frac{x_1 + x_2 + x_3 + x_4}{4} = \frac{\frac{x_2+x_3}{2} + \frac{x_1+x_4}{2}}{2}$$

This form makes clear that for trapezoid fuzzy intervals, e *is the real value in* $\widetilde{I}$ *minimizing the average distance to the bounds of the most possible values* $[x_2, x_3]$ *of* $\widetilde{I}$ *and the bounds of the support* (x_1, x_4)*, i.e. the values that are possible at all. In this sense,* e *can also be considered as a possibilistic error minimizing defuzzification method.*

2.2 Fuzzy Coalition Games

Definition 2.10 *A fuzzy coalition game* $(\mathcal{A}, \widetilde{v})$ *consists of a set of agents* $\mathcal{A}$*, a fuzzy characteristic function* $\widetilde{v} : 2^{\mathcal{A}} \mapsto \widetilde{\mathbb{R}}$*, and the membership function of the fuzzy quantities* $\widetilde{v}(C)$ *that might be interpreted as expectation of the common coalition profit that is to be distributed among its members.*

[1]Carlsson and Fuller introduced a weighted version [4] of e which allows for adjusting the importance of different possibility levels.

The worth $\widetilde{v}(C)$ of a fuzzy-valued coalition C is a fuzzy set of its possible real-valued coalitional profits, represents a possibility distribution of the real coalition value $v(C) \in \mathbb{R}$, and has at least one modal value. If, for a given fuzzy coalition game, the coalition value v(C) is equal to one modal value of C for all possible coalitions C, it is equivalent to a (deterministic) coalition game [2].

Definition 2.11 *A* fuzzy configuration $(\mathcal{C}, \widetilde{u})$ *consists of a (crisp) coalition structure* $\mathcal{C}$ *and fuzzy payoff distribution* $\widetilde{u} : \mathcal{A} \mapsto \widetilde{\mathbb{R}}$. $\widetilde{u}$ *is called* $\widetilde{\cong}$*- efficient to a degree of*

$$\mu_{eff^{\widetilde{\cong}}}(\widetilde{u}) := \widetilde{\bigwedge}_{C \in \mathcal{C}} \left\{ \sum_{a_i \in C}^{\oplus} \widetilde{u}(a_i) \widetilde{\cong} \widetilde{v}(C) \right\}$$

with fuzzy similarity relation $\widetilde{\cong}$*. For* $a \in \mathcal{A}$ *and fuzzy ranking operator* $\widetilde{\geq}$*, the degree of individual* $\widetilde{\geq}$*-rationality (*$\mu_{indrat^{\widetilde{\geq}}}(a)$*), and overall* $\widetilde{\geq}$*-rationality (*$\mu_{indrat^{\widetilde{\geq}}}(\widetilde{u})$*) of the payoff distribution* $\widetilde{u}$ *is defined as* $\widetilde{u}(a) \widetilde{\geq} \widetilde{v}(a)$ *and* $\widetilde{\bigwedge}_{a \in A}\{\mu_{indrat^{\widetilde{\geq}}}(a)\}$*, respectively.*

The fuzzy Shapley value stable payoff distribution, introduced in [10], is $\widetilde{\geq}_P$-rational as well as $\widetilde{\cong}_P$-efficient with degree 1 if the coalition values are normalized fuzzy intervals. For reason of efficient negotiation, we adopt the fuzzified bilateral Shapley value for stable payoff distribution among members of bilaterally formed fuzzy valued coalitions.

Definition 2.12 *The fuzzy Shapley value* $\widetilde{\sigma}(a)$ *of agent* $a \in \mathcal{A}$ *in a fuzzy game* $(\mathcal{A}, \widetilde{v})$ *is* $\widetilde{\sigma}(a) = \sum_{C \subseteq \mathcal{A}}^{\oplus} \frac{(|\mathcal{A}|-|C|)!(|C|-1)!}{|\mathcal{A}|!}(\widetilde{v}(C) - \widetilde{v}(C \setminus \{a\}))$.

The fuzzy bilateral Shapley value $\widetilde{\sigma}_b(C_1 \cup C_2, C_i, v)$, $C_i, i \in \{1, 2\}$ *of the bilateral coalition* $C_1 \cup C_2$ *is defined as the fuzzy Shapley value of* C_i *in the game* $(\{C_1, C_2\}, \widetilde{v})$*:*

$$\widetilde{\sigma}_b(C_1 \cup C_2, C_i, \widetilde{v}) := \frac{1}{2}\widetilde{v}(C_i) \oplus \frac{1}{2}(\widetilde{v}(C_1 \cup C_2) \ominus \widetilde{v}(C_k))$$

with $k \in \{1, 2\}, k \neq i$.

Since the uncertainty denoted by $\widetilde{v}(C_1)$, $\widetilde{v}(C_2)$ and $\widetilde{v}(C_1 \cup C_2)$ is now represented by the fuzzy bilateral Shapley value, it requires appropriate defuzzification of the payoff distribution at the end of the coalition negotiations to enable crisp side-payments among coalition members. If, for example, coalition $C_1 \cup C_2$ has been negotiated, the agents may determine the crisp coalition value $v(C_1 \cup C_2)$ from the actual costs and rewards after having carried out the agreed joint actions. However, the real coalition values $v(C_1)$ and $v(C_2)$, in general, remain unknown to the agents, hence need to be defuzzified. For this purpose, we use a modified

[2]In the following, we use the term "fuzzy game" instead of "fuzzy coalition game".

fuzzy bilateral Shapley value that uses coalition values of subcoalitions defuzzified by the possibilistic mean, which leaves the fuzziness of the resulting payoffs to that of the joint coalition value only.

Definition 2.13 *The fuzzy bilateral Shapley value of given fuzzy game* $(\mathcal{A}, \widetilde{v})$ *with defuzzified values of subcoalitions* $\widetilde{\sigma}_b^e(C_1 \cup C_2,\, C_i,\, v)$,$C_i, i \in \{1,2\}$ *in the bilateral coalition* $C_1 \cup C_2$ *is defined as the fuzzy Shapley value of* C_i *in the game* $(\{C_1 \cup C_2, C_1, C_2\}, \widetilde{v})$*. With* $k \in \{1,2\}, k \neq i$*,*

$$\widetilde{\sigma}_b^e(C_i, \widetilde{v}) := \frac{1}{2}e(\widetilde{v}(C_i)) \oplus \frac{1}{2}(\widetilde{v}(C_1 \cup C_2) \ominus e(\widetilde{v}(C_k)))$$

Similar to the crisp bilateral Shapley value, we use a recursive payoff distribution of the modified fuzzy bilateral Shapley value based on the recursively bilateral formation of coalition structures. Each agent maintains its individual coalition history tree in due course of the bilateral negotiations it participates in as member of a coalition.

Definition 2.14 *The fuzzy payoff distribution* $\widetilde{u}$ *within any bilateral coalition* C *in a fuzzy game* $(\mathcal{A}, \widetilde{v})$ *is called* recursively fuzzy bilateral Shapley value stable *iff for every non-leaf node* C^* *of the coalition history tree* T_C : $u(C_i^*) = \widetilde{\sigma}_b^e(C^*,\, C_i^*,\, \widetilde{v}_{C^*}), i \in 1,2$ *with* $\forall C^{**} \subseteq \mathcal{A}$*:*

$$\widetilde{v}_{C^*}(C^{**}) = \begin{cases} \widetilde{\sigma}_b^e(C^p,\, C_k^p,\, \widetilde{v}_{C^p}) & \text{if } C^p \in T_C, \\ & C^* = C_k^p, k \in 1,2 \\ \widetilde{v}(C^{**}) & \text{otherwise} \end{cases}$$

For each coalition $C \subseteq \mathcal{A}$ *we define the fuzzy* local worth *of individual agent* $a \in C$ *as* $\widetilde{lworth_C}(C^*) := \sum_{a \in C} \widetilde{lworth}(C^*)$ *with* $C^* \subseteq \mathcal{A}, C \subseteq C^*$*.* $\widetilde{lworth_a}(C)$ *denotes the fuzzy gain of* a *for accomplishing its tasks in* C *on behalf of its user or other agents in* C*, including costs.*

Each coalition value is the sum of the local worth of each of its members $\widetilde{v}(C) = \sum_{a \in C}^{\oplus} \widetilde{lworth_a}(C)$. Further, the expected gain in utility of any potential bilateral coalition candidate is the difference between what it may expect to obtain in the coalition merger in terms of the bilateral Shapley value and its expected self-value.

Definition 2.15 *For a fuzzy game* $(\mathcal{A}, \widetilde{v})$*, the bilateral Shapley value based expected utility gain of a subcoalition* C *in the coalition* $C \cup C^*, C, C^* \subset \mathcal{A}$ *is*

$$\widetilde{g}_{\widetilde{v}}(C, C \cup C^*) := \widetilde{\sigma}_b^e(C \cup C^*, C, \widetilde{v}) - e(\widetilde{v}(C))$$

Lemma 2.16 *Let a fuzzy game* $(\mathcal{A}, \widetilde{v})$ *and* $C_1, C_2 \subset \mathcal{A}$*. Then* $\widetilde{g}_{\widetilde{v}}(C_1, C_1 \cup C_2) = \widetilde{g}_{\widetilde{v}}(C_2, C_1 \cup C_2)$

Proof: By definitions 2.15 and 2.13, and because of the properties of $\oplus$ and $\ominus$ when applied to at least one crisp operand discussed e.g. in [7], we can rewrite

$$\begin{aligned}
&\widetilde{g}_{\widetilde{v}}(C_1, C_1 \cup C_2) \\
&= \frac{1}{2}e(\widetilde{v}(C_1)) \oplus \frac{1}{2}(\widetilde{v}(C_1 \cup C_2) \ominus e(\widetilde{v}(C_2))) \ominus e(\widetilde{v}(C_1)) \\
&= \frac{1}{2}\widetilde{v}(C_1 \cup C_2) \ominus \frac{1}{2}e(\widetilde{v}(C_2)) \ominus \frac{1}{2}e(\widetilde{v}(C_1)) \\
&= \frac{1}{2}\widetilde{v}(C_1 \cup C_2) \ominus \frac{1}{2}e(\widetilde{v}(C_1)) \ominus \frac{1}{2}e(\widetilde{v}(C_2)) \\
&= \widetilde{g}_{\widetilde{v}}(C_2, C_1 \cup C_2)
\end{aligned}$$

□

3 Fuzzy-Valued Stable Coalition Negotiation

Algorithm 3.1 *(***BSCA-F***).*
Given $\mathcal{A}$*, initial configuration* $(\mathcal{C}_0, \widetilde{u}_0)$ *with singleton sets in* $\mathcal{C}_0$ *and* $\widetilde{u}_0(a) = \widetilde{v}(a)$*, fuzzy ranking operator* $\widetilde{\circ} \in \{\widetilde{\geq}_P, \widetilde{\geq}_N, \widetilde{>}_P, \widetilde{>}_N\}$*, ranking threshold* t*, and negotiation round counting variable* $r := 1$*. Further, each coalition determines its representative* Rep_C *via voting; representatives are ranked according to given ascending order* $o : \mathcal{A} \mapsto \mathbb{N}$ *of agents based on, for example, available computational ressources.*
Each agent $a \in C \in \mathcal{C}_r$ *performs:*

1. *Communication:* If $a \neq Rep_C$ then go to step 3f; else do for all $C^* \in \mathcal{C}_r, C^* \neq C$:
   (a) send $\widetilde{lworth_C}(C \cup C^*)$ to Rep_{C^*}
   (b) receive $\widetilde{lworth_{C^*}}(C \cup C^*)$ from Rep_{C^*}
   (c) compute $\widetilde{v}(C \cup C^*) = \widetilde{lworth_C}(C \cup C^*) \oplus \widetilde{lworth_{C^*}}(C \cup C^*)$
2. *Proposal Generation*
   (a) $Cand_C := \left\{C^* \,\middle|\, C \in \mathcal{C} \setminus C,\, (\widetilde{g}(C, C \cup C^*, \widetilde{v})\widetilde{\circ}\widetilde{0}) \geq t\right\}$
   (b) If $Cand_C \neq \emptyset$, send proposal to Rep_{C^+} of most beneficial C^+ to form joint coalition $C \cup C^+$. In case of multiple possible choices, uniquely select best representative $o(Rep_{C \cup C^+}) = \min\{o(Rep_{C^*}) \mid \widetilde{g}_{\widetilde{v}}(C, C \cup C^+) \in \max^{\widetilde{\circ}}\{\widetilde{g}_{\widetilde{v}}(C, C \cup C^{**}) \mid C^{**} \in Cand_C\}\}$
   (c) Receive all proposals from all other Rep_{C^*}, $C^* \in \mathcal{C}_r, C^* \neq C$
3. Coalition Forming
   (a) Set $New := \emptyset$ and $Obs := \emptyset$
   (b) If a proposal was sent to as well as received from C^+ , form joint coalition $C \cup C^+$:

i. If $o(Rep_C) < o(Rep_{C^+})$ then $Rep_{C \cup C^+} := Rep_C$ else $Rep_{C \cup C^+} := Rep_{C^+}$

ii. inform all other Rep_{C^*}, $C^* \in \mathcal{C}_r, C^* \neq C, C^* \neq C^+$ and all $a^* \in C$, $a \neq Rep_C$ about the newly formed coalition and $Rep_{C \cup C^+}$

iii. $New := \{C \cup C^+\}$, $Obs := \{C, C^+\}$, $Cand_C := \emptyset$

(c) Receive all messages about new coalition. For each new coalition $C_1 \cup C_2$ and $Rep_{C_1 \cup C_2}$ do: $Cand_C := Cand_C \setminus \{C_1, C_2\}$, $New := New \cup \{C_1 \cup C_2\}$and $Obs := Obs \cup \{C_1, C_2\}$.

(d) If no new coalition was formed, go to step 2b.

(e) Send the sets New and Obs to all other coalition members $a^* \in C$, $a \neq Rep_C$

(f) If $a \neq Rep_C$ then receive sets New and Obs from Rep_C.

(g) Set $r := r + 1, \mathcal{C}_r := (\mathcal{C}_{r-1} \setminus Obs) \cup New$, and u_r according to recursive fuzzy bilateral Shapley value based on the coalition structures $\mathcal{C}_r \dots \mathcal{C}_0$.

(h) If $C_r = C_{r-1}$then stop; else go to step 1

Proposition 3.2 *Between step 2.b and 3.c in any round $r \in \mathbb{N}$, the coalition $C_1 \cup C_2$, $C_1, C_2 \in \mathcal{C}_r$, which is among the overall most profitable coalitions in the sense that $\widetilde{g}_{\widetilde{v}}(C_1, C_1 \cup C_2) \in \max^{\widetilde{\circ}} \{\widetilde{g}_{\widetilde{v}}(C, C \cup C^{**}) \mid C \in \mathcal{C}_r, C^{**} \in Cand_C\}$, and $o(Rep_{C_1 \cup C_2})$ is minimal as compared to o of other overall most profitable coalitions, is formed, or no proposals are sent at all.*

Proof: Because of lemma 2.16, we have that if $C_1 \cup C_2$ is in the set $Cand_{C_1}$, it is also in the set $Cand_{C_2}$. From the properties of $\widetilde{\geq}_P, \widetilde{\geq}_N, \widetilde{>}_P$ and $\widetilde{>}_N$ discussed in [6], it is clear that for a set of fuzzy quantities X, if $F_1 \in X$: $F_1 \in \max^G X$, then also $F_1 \in \max^G Y \subseteq X$ with $F_1 \in Y$. Further, $\widetilde{g}_{\widetilde{v}}(C_1, C_1 \cup C_2) = \widetilde{g}_{\widetilde{v}}(C_2, C_1 \cup C_2)$ because of lemma 2.16. Thus, with (a) it follows that $\widetilde{g}_{\widetilde{v}}(C_1, C_1 \cup C_2) \in \max^{\widetilde{\circ}} \{\widetilde{g}_{\widetilde{v}}(C_i, C_i \cup C^{**}) \mid C^{**} \in Cand_{C_i}\}$ for both $i = 1$ and $i = 2$. With the unambiguousness of the agent ordering o and (b), it is then clear that in step 2.b C_1 and C_2 send proposals to each other and thus form $C_1 \cup C_2$ in step 3.c. □

Lemma 3.3 *In round $r \in \mathbb{N}$, the iteration between step 2.b and 3.d is done at most $\frac{|\mathcal{C}_r|}{2}$ times by each agent.*

Proof: Assume there have been $k \in \mathbb{N}$ iterations in a given round of the BSCA-F and $l \in \mathbb{N}$ new coalitions have been formed. Then proposition 3.2 implies that $k \leq l$. Step 3.b.iii implies that every coalition can merge with another one only once in each round of the BSCA-F, and thus limits the overall number of new coalitions per round to at most $\frac{|\mathcal{C}_r|}{2}$. So we have $k \leq l \leq \frac{|\mathcal{C}_r|}{2}$. □

Lemma 3.4 *The BSCA-F terminates after at most $|\mathcal{A}|$ rounds.*

Proof: In each non-final round $r \in \mathbb{N}$ of the BSCA-F at least one new coalition may form, i.e. $|\mathcal{C}|_{r+1} \leq |\mathcal{C}|_r - 1$. Thus, after $|\mathcal{A}| - 1$ rounds, we have $|\mathcal{C}|_{|\mathcal{A}|-1} \leq 1$, which means that the BSCA-F terminates in round $|\mathcal{A}|$. □

Theorem 3.5 *The worst-case runtime of the BSCA-F for each agent is in $O(|\mathcal{A}|^4)$ assuming constant time for operations on fuzzy quantities.*

Proof: In step 2.b, each C has to find the (crisp) maximum set of the fuzzy gains for coalitions in $Cand_C$, with $|Cand_C| \leq |\mathcal{C}_r|$. From definitions 2.6 and **??** it follows that this can be done in $O(|\mathcal{C}_r|^2)$. Because all other individual operations are of less complexity and with lemma 3.3, the iteration between step 2.b and 3.d thus is in $O(|\mathcal{C}_r|^3)$. Since $|\mathcal{C}_r| \leq |\mathcal{A}|$ and lemma 3.4, the overall runtime of the BSCA-F is then $O(|\mathcal{A}|^4)$. □

Theorem 3.6 *The total number of messages sent by agents using the BSCA-F for coalition negotiation is $O(|\mathcal{A}|^2)$.*

Proof: In each round $r \in \mathbb{N}$, each representative of a coalition C sends $|\mathcal{C}_r| - 1$ messages in step 1.a, a single proposal message in 2.b, at most one time $|\mathcal{C}_r| - 2$ messages in step 3.b.ii and $|C| - 1$ messages in step 3e. So the number of messages per representative per round is bound by $|\mathcal{C}| \leq |\mathcal{A}|$. The number of messages sent by the $|\mathcal{A}| - |\mathcal{C}|$ non-representatives is zero. So with lemma 3.4, the overall number of messages sent is lower or equal $|\mathcal{A}|^2$. □

Coalition negotiation using the BSCA-F yields a coalition structure $\mathcal{C}$ with recursively fuzzy bilateral Shapley value stable payoff distribution. Since theses fuzzy payoffs originate from the fuzzy coalition values of coalitions in $\mathcal{C}$ only (all other fuzzy coalition values are defuzzified by use of the possibilistic mean value) we have to defuzzify the values of exactly these coalitions in $\mathcal{C}$ only. It appears plausible that the real coalition values become known to the corresponding coalition members after their coalitions have been formed and contracted actions are carried out. Otherwise, we may also use the possibilistic mean to derive at least a reasonable expectation of them. Thus, in both cases, we may obtain crisp coalition values for all coalitions in $\mathcal{C}$, hence crisp payoffs.

4 Example Application

4.1 Definition of the Game

In the following, we demonstrate how the BSCA-F could be applied to negotiate economically rational coalitions of

Cat.	M_1	M_2	M_3	M_4
a)	politics	column	column 1	photo mag
b)	feature sect.	travel mag	column 2	feature sect.

Table 1. Content provided by magazines

I_1	$\widetilde{A}^1_.$	I_2	$\widetilde{A}^2_.$
M_2 a)	$\widehat{(2.4, 3.6, 4.2, 4.8)}$	M_1 a)	$\widehat{(8.4, 12, 14.4, 16.8)}$
M_2 b)	$\widehat{(1.08, 1.2, 1.56, 1.8)}$	M_3 a)	$\widehat{(6.6, 7.2, 7.8, 8.4)}$
M_3 a)	$\widehat{(9.6, 12, 14.4, 15.6)}$	M_4a)	$\widehat{(6, 9, 9.333, 10)}$
M_4 a)	$\widehat{(3.6, 7.2, 12.8, 16.4)}$		
I_3	$\widetilde{A}^3_.$	I_4	$\widetilde{A}^4_.$
M_2 b)	$\widehat{(13.2, 14.4, 15, 15.6)}$	M_1 b)	$\widehat{(3.6, 3.72, 4.08, 4.2)}$
		M_2 b)	$\widehat{(7.2, 7.56, 10, 12.5)}$

Table 2. Additional income per category

online magazines in the Internet. Consider four online magazines, $M_1 - M_4$, that are interested in exchanging content for different reasons such as customer recruitment. Suppose that each of them is reluctant to provide more content to potential partners than it would obtain in return for a certain payoff, hence agrees to contribute only two categories of the content to the coalition it participates in table 1). Content is provided to coalition partners on a daily basis, whereas coalition contracts in total will hold for one year after which negotation may be restarted. To prevent antitrust matters, coalitions with more than three members are ruled out. Each magazine M_i is represented by an agent a_i which carries out negotiation on behalf of M_i. Each magazine M_i would publish only such content provided by coalition partners which is in line with the general style of M_i. The set of categories M_i is interested in is called I_i. For each $x \in I_i$, M_i fuzzily estimates the number $\widetilde{A}^i_x$ of additional accesses for one year it can achieve by publishing x. For simplicity, we assume these estimations are independent of each other. The sets I_i and estimations $\widetilde{A}^i_x$ (in thousands) are given in table 2. Any single access to content of a magazine M_i, $1 \le i \le 4$, is subject to a given price P_i (in Euros) determined by M_i. The prices are $P_1 := 2$, $P_2 := 1.5$, $P_3 := 1.8$ and $P_4 := 2.0$. The additional income produced by a magazine M_i by coalescing with a magazine M_k is $\sum^{\oplus}_{M_k x) \in I_i} \widetilde{A}^i_{M_k x)} \odot P_i$, and the total additional income $\widetilde{ai}_i(C)$ for M_i in coalition C is given with $\widetilde{ai}_i(C) = \sum^{\oplus}_{M_k \in C, k \neq i} \sum^{\oplus}_{M_k x) \in I_i} \widetilde{A}^i_{M_k x)} \odot P_i$ M_1 and M_2 arguably have the best cooperation opportunities in this game. On the cost side, we consider only volume-based transfer costs (in Euros) with given transfer price T_i per MB depending on the internet connection of magazine M_i. The costs are $T_1 := 0.02$, $T_2 := 0.01$, $T_3 := 0.025$ and $T_4 := 0.012$. Based on experiences in the past, each M_i estimates the amount of data V_x it would need to transfer

	$V_{M_. a)}$	$V_{M_. b)}$
M_1	$\widehat{(12, 18, 24, 42)}$	$\widehat{(6, 12, 30, 60)}$
M_2	$\widehat{(1.2, 3.6, 6, 7.2)}$	$\widehat{(96, 120, 156, 204)}$
M_3	$\widehat{(3.6, 8.4, 12, 14.4)}$	$\widehat{(2.4, 3.6, 4.2, 5.4)}$
M_4	$\widehat{(180, 192, 204, 216)}$	$\widehat{(18, 24, 30, 36)}$

Table 3. Amount of category data (in 100MB)

C	$\widetilde{v}(C)$
a_1, a_2	$\widehat{(38060, 48552, 60739, 70454)}$
a_1, a_3	$\widehat{(18835, 23789, 28674, 31128)}$
a_1, a_4	$\widehat{(13338, 20976, 33022, 40552)}$
a_2, a_3	$\widehat{(32967, 36185, 38293, 40370)}$
a_2, a_4	$\widehat{(19010, 22932, 32981, 39114)}$
a_3, a_4	$\widehat{(-815, -751, -684, -612)}$
a_1, a_2, a_3	$\widehat{(89564, 108332, 127576, 141865)}$
a_1, a_2, a_4	$\widehat{(69646, 91830, 126204, 149649)}$
a_1, a_3, a_4	$\widehat{(30690, 43458, 60529, 70642)}$
a_2, a_3, a_4	$\widehat{(50331, 57648, 69967, 78328)}$
others	$(\widehat{0})$

Table 4. Coalition values (rounded)

per category x during one year as shown in table 3. Every magazine M_i has to pay for both incoming and outgoing traffic, means $(V_{M_i a)} \oplus V_{M_i b)} \oplus V_{M_k a)} \oplus V_{M_k b)}) \odot T_i$ for data transmitted to/from each coalition partner M_k. Hence, the total cost $\widetilde{c}_i(C)$ for M_i in coalition C is $\widetilde{c}_i(C) = \sum^{\oplus}_{M_k \in C, k \neq i} (V_{M_i a)} \oplus V_{M_i b)} \oplus V_{M_k a)} \oplus V_{M_k b)}) \odot T_i$. Having both total additional income and costs for each magazine M_i in a coalition C, we obtain their local worth individually by $\widetilde{lworth}_{a_i}(C) = \widetilde{ai}_i(C) \ominus \widetilde{c}_i(C)$, and resulting fuzzy coalition values (cf. table 4.

4.2 Negotiation with the BSCA-F

For this example, we select the necessity of dominance $\widetilde{\geq}_N$ as fuzzy ranking operator, and $o(a_i) := i$ for agent ordering. In the first round, in step 2a, all coalitions prefer each other except of $a_3 \cup a_4$ which is clearly a non-profitable coalition. Both a_1 and a_2 mutually propose $a_1 \cup a_2$ to each other as the most profitable joint coalition with payoff half of the coalition value: $\widetilde{u}(a_1) = \widetilde{u}(a_2) = \frac{1}{2}\widetilde{0} \oplus \frac{1}{2}(\widehat{(38060, 48552, 60739, 70454)} \ominus \widetilde{0}) = \widehat{(19030, 24276, 30369.5, 35227)}$. In the second round, $a_1 \cup a_2$ sends a proposal to a_3 rather than a_4 according to $\widetilde{\geq}_N$ with $e(\widetilde{v}(\{a_1, a_2\}) = \frac{38060+48552+60739+70454}{4} = 54451$, $\widetilde{g}_{\widetilde{v}}(a_1 \cup a_2, (a_1 \cup a_2) \cup a_3) = \frac{1}{2}(\widetilde{v}(\{a_1, a_2, a_3\}) \ominus e(\widetilde{v}(\{a_1, a_2\}) \ominus e(\widetilde{v}(\{a_3\}))$

$= \frac{1}{2}((89564, 108\widehat{332, 127}576, 141865) \ominus 54451 \ominus 0)$ $= (17556, 269\widehat{40, 36}562, 43706) = \widetilde{g}_{\widetilde{v}}(a_3, (a_1 \cup a_2) \cup a_3)$ (cf. lemma 2.16). Similarily, $\widetilde{g}_{\widetilde{v}}(a_1 \cup a_2, (a_1 \cup a_2) \cup a_4)$ $= \widetilde{g}_{\widetilde{v}}(a_4, (a_1 \cup a_2) \cup a_4) = (7597, 186\widehat{89, 35}876, 47599)$ Please note that with $>_P$, the choice would have been a_4. The payoff of the new coalition is distributed as follows: $\widetilde{u}(a_1) = \widetilde{u}(a_2) = \widetilde{\sigma}^e_b(a_1 \cup a_2,\, a_1,\, \widetilde{\sigma}^e_b(C^*,\, a_1 \cup a_2,\, \widetilde{v}(C^*)))$ $= \frac{1}{2}(\frac{1}{2}e(\widetilde{v}(a_1 \cup a_2)) \oplus \frac{1}{2}((89564, 108\widehat{332, 127}576, 141865)$ $\ominus 0)) = (17556, 269\widehat{40, 36}562, 43706)$. In the third round, the BSCA-F terminates, since the value of the grand coalition is zero, thus is not a candidate for anyone.

4.3 Defuzzification

Since coalition contracts, in this example, are valid for one year, there are two options to determine the real coalition values: Either wait for one year and then analyze the additional income and the costs that were realized in the period; or defuzzify the coalition values just when negotiations are finished, using the possibilistic mean value. Due to space limitations, we only discuss the second case for the coalition $C^* := \{a_1, a_2, a_3\}$ formed, and the possibilistic mean value of $\widetilde{v}(C^*)$: $e(\widetilde{v}(C^*)) = \frac{89564+108332+127576+141865}{4} = 116834$. Computing the recursive bilateral Shapley value with $e(\widetilde{v}(C^*))$, we obtain $u(a_1) = u(a_2) = 42821$, which is equal to $e(\widetilde{u}(a_1))$ ($= e(\widetilde{u}(a_2))$ due to the additivity of e. Similarily, it holds that $u(a_3) = e(\widetilde{u}(a_3)) = 31192$. To summarize, when negotiations are finished, the agent have to compute the possibilistic mean values of the fuzzy payoffs only. this yields the same result as if they would compute the recursively bilateral Shapley value stable payoffs for the possibilistic mean of the coalition values. The resulting payoffs appear to be intuitively sound, since a_1 and a_2, the agents with most beneficial cooperation opportunities, are assigned more payoff than a_3. Further, let us consider computing the fuzzy payoffs by recursively applying the non-defuzzyfying fuzzy bilateral Shapley value as defined in **??** instead. This means that the fuzzy payoffs now also contain the fuzzyness of the values of the subcoalitions. Then we obtain the fuzzy payoffs $\widetilde{u}^*(a_1) = \widetilde{\sigma}_b(a_1 \cup a_2,\, a_1,\, \widetilde{\sigma}_b(C^*,\, a_1 \cup a_2,\, \widetilde{v}(C^*)))$ $= (31906, 392\widehat{21, 47}079, 53080)$ ($= \widetilde{u}^*(a_2)$) and $\widetilde{u}^*(a_3)$ $= \widetilde{\sigma}_b(C^*,\, a_3,\, \widetilde{v}(C^*))) = (9555, 237\widehat{97, 39}512, 51903)$.

5 Conclusions

We presented a new low-complexity coalition forming algorithm, BSCA-F, that enables agents to negotiate bilateral Shapley value stable coalitions in uncertain environments, and demonstrated it by example. In particular, we showed that utilizing the possibilistic mean value for defuzzifying negotiated fuzzy agent payoffs appears reasonable. However, the choice of the fuzzy ranking operator is supposed to be equal for each agent; future work includes relaxation of this requirement.

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