Chapter 31
Math-Bridge: Closing Gaps in European Remedial Mathematics with Technology-Enhanced Learning

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Abstract Math-Bridge is an e-Learning platform for online courses in mathematics. It has been developed as a technology-based educational solution to the problems of bridging courses taught in European universities. Math-Bridge has a number of unique features. It provides access to the largest in the World collection of multilingual, semantically annotated learning objects (LOs) for remedial mathematics. It models students’ knowledge and applies several adaptation techniques to support more effective learning, including personalized course generation, intelligent problem solving support and adaptive link annotation. It facilities a direct access to LOs by means of semantic search. It provides rich functionality for teachers allowing them to manage students, groups and courses, trace students’

† Dr. Erica Melis was the initiator and the coordinator of the Math-Bridge project. She passed away during the implementation of Math-Bridge.
Math-Bridge offers a complete solution for organizing technology-enhanced learning (TEL) of mathematics on individual-, course- and/or university level.

1 Introduction

Many students enrolling into European colleges and universities lack mathematical competencies necessary for their studies, especially, in math-intensive engineering and science disciplines (ACME 2011). This often leads to serious learning problems, and even causes students to drop out of their learning programs. For instance, drop-out rates for most engineering disciplines in Germany grew about 10% over the last 15 years and now stay at 25-35%. Drop-out rates for math-intensive science study programs in German universities and colleges have also grown across the board to 15-40% (Heublein et al. 2008). Similar figures apply for several other countries of EU, such as the Netherlands, Germany, Spain, UK, etc.

One source of the problem is that many students simply underestimate the requirements to their mathematical background when selecting a study program. This often results in confusion, frustration and lack of motivation to continue the study. On the other hand, the schools themselves cannot always provide skilful teachers and enough content to prepare their pupils for the university-level mathematics courses. Although high drop-out rates cannot only be solved by focusing on mathematics education, an early opportunity to close the competency gap and increase students’ awareness about the of university programs requirements can help them to make an informed choice of future studies, prepare properly and avoid potential detrimental effects on motivation.

In order to facilitate pragmatic efforts in this direction, and take a significant step towards improving European educational practices in the field of remedial mathematics, in 2009, a consortium of educators, mathematicians and computer scientists from nine universities and seven countries initiated the project Math-Bridge1. This paper presents the technical side of this project. Section 2 underlines the problems of existing remedial courses offered by individual European universities. Section 3 briefly summarizes the most important aspects of the approach implemented by Math-Bridge. Sections 3 to 6 provide the details of the Math-Bridge technology including the design and implementation of the developed e-learning platform, characteristics of the accumulated collection of digital mathematical content and the main functionality available to the user of Math-Bridge, both students and teachers. Section 7 concludes the paper.

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1 This work has been supported by the EU eContentplus project “Math-Bridge: European Remedial Content for Mathematics” (ECP-2008-EDU-428046).
2 Problems of existing remedial courses

Educators across EU have long realized this set of challenges. The common solution is offering to students a dedicated bridging (or remedial) course in the beginning of their study program. Although these courses might help to remedy the problem in the universities administering them, there are several drawbacks that prevent implementation of a broader, cheaper and more effective solution.

Accessibility
To improve the situation, access to bridging mathematical content has to be provided not only on paper during remedial sessions at the university, but constantly on the Web. When (or even before) choosing a university/college, students should be able to prepare for the requirements of their study subjects.

Support for cross-cultural and multi-lingual access
The target competencies that students need to master often vary in description and names across countries, study programs and universities. It is not easy for prospective students, to get access to such requirements and learn them in advance. A solution is necessary that would provide a mapping between different institutions’ curricula/requirements to a more generic set of competencies.

At the same time, mathematical notations themselves often vary across countries, which makes understanding of math content much more difficult for international students. Naturally, this problem also occurs when the content is available only in one language. With student mobility increasing and Bologna process gaining momentum, the number of international students in European universities grows. For international students, transition to the university-level mathematics is aggravated by the necessity to learn it in a foreign language that they did not use when taking the mathematics courses in a school (Setati et al. 2011).

A related issue is the lack of appropriate metadata that would otherwise facilitate the discovery of learning content in a desired language.

Content reuse and interoperability
Even when the remedial content for mathematics is digitized, it lacks the most basic support for content discoverability and interoperability. This means, the content cannot be easily found, altered and reused by anybody but its authors. Content authoring for educational systems is a tedious and expensive procedure. Therefore, it is crucial to ensure that the high-quality remedial content becomes easily available for the third-party users.

The problems often start with the formats in which this content exists. Many teachers simply distribute their course materials as Word or TeX-documents, thus supporting no basic means for content reuse. Moreover, a large part of the content is compartmentalized by publishers or universities. It is not annotated with proper metadata, i.e. does not carry the semantic (mathematical) information that would be necessary to discover and retrieve the appropriate LOs easily.

In order to realize the full potential of WWW as the infrastructure for content delivery, the remedial mathematical content should be:
1. Implemented using standard-based semantic representation (XML and RDF-based formats provide the basic means for content reuse across systems and contexts) (Buswell et al. 2004; Carlisle et al. 2010; Kohlhase 2006);
2. Dissected into individual LOs (this way, meaningful pieces of learning material can be discovered, and reused, the content can be re-assembled according to the curricular of different courses, intelligent e-learning applications can present to the students only the most appropriate learning resources);
3. Provided with open-standard metadata, e.g. (IEEE 2002; IMS 2003; ADL 2009) (which will facilitate content discovery, and enable adaptive access to the content based on its characteristics and the learning needs of the student);
4. Authored in such a way that the meaning of mathematical symbols is decoupled from their rendering in the browser (this is essential for supporting multi-lingual and multi-cultural access to the content).

**Interactivity**

To support meaningful learning experiences and enhance students’ engagement, remedial content collections must have interactive LOs that can react to students’ actions by providing proper feedback. Unfortunately, the dominant type of learning content available online is instructional texts and lecture slides.

**Infrastructures for students, teachers, and authors**

Students’ motivation, self-assessment, and performance can be improved by implementing a range of TEL approaches supporting adaptive access to LOs, timely assessment of student knowledge, self-organized and exploratory learning. This functionality is not available for them within the current bridging courses.

An adequate infrastructure for effective authoring of learning resources is also missing. Developing an individual LO or assembling an entire course remains a very complex task poorly supported by the majority of existing authoring tools.

A teacher of a bridging course must be able to easily manage all key aspects of the course, including student, course material, assessment tests, etc. A teacher should be also provided with rich facilities for monitoring students’ progress, detecting potential learning problems and intervening if necessary.

**3 Math-Bridge: the Approach**

The Math-Bridge project has addressed the outlined set of problems by applying a range of techniques from the fields of Intelligent Tutoring Systems, Adaptive Hypermedia, Semantic Web and TEL. The result of these efforts is the e-Learning platform for bridging courses with a number of unique features. It provides multi-lingual and multi-cultural semantic access to remedial mathematics content, which adapts to the requirements of a learner and his/her subject of study. It brings together content from different European sources and offers it in a unified way. This access is provided through an online Pan-European learning service for remedial mathematics, which is built by collecting appropriate learning resources,
extending them in terms of structure and multi-linguality and making them useful and easy-to-find. The extended formats of the content makes a wider use of standards and, hence, enables content reusability and transferability between different learning environments. In order to achieve this, an analysis of the target bridging competencies required for target subjects of study has been performed, the semantic and multi-lingual search software has been developed, the assessment tools have been implemented, and a range of remedy-scenario has been authored, which includes specific diagnostic means and decisions for the transition from school to higher education. The solution implemented by Math-Bridge has been based on achieving several operational objectives:

1. To collect and harmonize high-quality remedial content developed by experts in bridging-level mathematics and make this content Web-accessible;
2. To enable cross-cultural and multi-lingual presentation of this content, thus, promoting its reuse across the borders;
3. To motivate the technological reuse of the content by implementing it in a shareable format and enriching it with metadata based on open standards;
4. To offer different types of personalized access to the content, thus supporting multiple usage scenarios of the platform: from individual exploratory e-Learning to classroom-based knowledge training and testing;
5. To foster the platform adoption by increasing its usability not only for students, but also for other stakeholders, including teachers and university officials.

4 Math-Bridge: Content and Knowledge Base

Mathematical Content Collections
The Math-Bridge content base consists of several collections of learning material covering the topics of secondary and high school mathematics. They were originally developed for teaching bridging courses by mathematics educators from several European universities participating in the project.

The content collections have been transformed through a sequence of operations in order to enable discoverability, interoperability and adaptability of LOs constituting them. Fig. 31.1 presents the complete procedure of content transformation, step-by-step. The resulting database of remedial content is available as a collection of individual LOs, transformed into the OMDoc format for mathematical documents (Kohlhase 2006) and provided with metadata.

Compared to the majority of adaptive e-Learning applications, Math-Bridge supports a multitude of LO types. The OMDoc language used for representing content in Math-Bridge defines a hierarchy of LOs to describe the variety of mathematical knowledge. On the top level, LOs are divided into concept objects and satellite object. Satellite objects are the main learning activities, they structure the learning content, which students practice with: exercises, examples, and instructional texts. Concept objects have a dualistic nature: they can be physically
presented to a student, and s/he can browse them and read them; at the same time
they are used as elements of domain semantics, and, as such, employed for representing knowledge behind satellite objects and modelling students’ progress.

There are five main types of concept object available in Math-Bridge (see Fig. 31.2). Symbol is a special kind of concept objects. They represent the most abstract entities in Math-Bridge, atomic mathematical concepts, which do not have content of their own. A symbol has a representation – a physical manifestation, which can be shown to a student and used in equations, but there is no actual learning content behind a symbol. Math-Bridge symbols are combined into an ontology, which is described on page 444.

The rest of the concept objects model the most typical mathematical notions:

- Definition is a statement, indicating meaning of one or several symbols;
- Axiom is a postulate about one or several symbols;
- Assertion is a statement about symbols; there can be several types of assertions, such as theorem, lemma, and corollary;
- Proof represents a formal inference of an assertion and is always connected to the assertion it proves.

The total number of LOs in the Math-Bridge content base is almost 11,000. Table 31.1 presents the details of this content base broken down by LO type.
Fig. 31.2 Hierarchy of LO types in Math-Bridge

Table 31.1 Different types of LOs in the Math-Bridge content collection

<table>
<thead>
<tr>
<th>Type</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interactive Exercise</td>
<td>5060</td>
</tr>
<tr>
<td>Instructional Text</td>
<td>2142</td>
</tr>
<tr>
<td>Learning Example</td>
<td>1563</td>
</tr>
<tr>
<td>Concept Definition</td>
<td>950</td>
</tr>
<tr>
<td>Assertion</td>
<td>634</td>
</tr>
<tr>
<td>Mathematical Proof</td>
<td>334</td>
</tr>
<tr>
<td>Axiom</td>
<td>36</td>
</tr>
</tbody>
</table>

Metadata Schema

The content collections of Math-Bridge consist of multiple LOs of different types. Both, the collections and the individual LOs are described in terms of their properties and attributes. The LOs link to each other by multiple relations. Both, the attributes describing LOs and the relations linking them together are specified via various metadata. The metadata elements can be divided into the following three categories:

- descriptive metadata used for administrative, cataloguing and licensing purposes; represented mainly using the Dublin Core standard (DCMI 1999).
- pedagogical metadata helping authors to specify multiple educational properties of LOs; adopted and extended from IEEE LOM and IMS LD standards (IEEE 2002; IMS 2003).
- semantic metadata connecting different LOs to one another; partially relaying on OMDoc and SKOS standards (Kohlhase 2006; Miles and Bechhofer 2009).

Math-Bridge metadata plays the core role in the overall architecture of the platform. It enables LOs discovery, course composition, students’ knowledge tracing and subsequent adaptation of the learning content.

Math-Bridge Ontology

In order to model the domain of bridging mathematics, an ontology for the target subset of mathematical knowledge has been created. It serves as a reference point
for all content collections and provides the source of the most abstract semantic metadata encoded through symbols. The ontology defines more than 600 concepts. It is used by the system logic for modelling students’ knowledge, and adaptive course generation. The ontology is available in OMDoc and OWL² (McGuinness and van Harmelen 2004).

Multilingual/Cross-cultural Aspects and Notation Census
Math-Bridge content is available in seven languages: English, German, French, Spanish, Finnish, Dutch and Hungarian. The user can specify the language, in which s/he would like to read the content. To support multilingual students, individual LOs can be translated on the fly. It is important to mention, that Math-Bridge translates not only the text but also the presentation of formulæ. Although mathematics is often called a “universal language”, this is not fully true. In many countries, the same mathematical concepts use very different symbols (Libbrecht 2010). In order to address this challenge, Math-Bridge separates the semantic and the presentation layer of math symbols. Inside the content, symbols are encoded using unambiguous entities, and when presented to the user, a correct notation is chosen based on the current language (Melis et al. 2009). A public “notation census” has been conducted to document different notations of all symbols in all languages³. Table 31.2 presents the quantities of LO translations available for each language.

Table 31.2 Different types of LOs in the Math-Bridge content collection

<table>
<thead>
<tr>
<th>Language</th>
<th>Count</th>
</tr>
</thead>
<tbody>
<tr>
<td>English</td>
<td>9792</td>
</tr>
<tr>
<td>German</td>
<td>9792</td>
</tr>
<tr>
<td>Spanish</td>
<td>5099</td>
</tr>
<tr>
<td>Dutch</td>
<td>5484</td>
</tr>
<tr>
<td>French</td>
<td>4391</td>
</tr>
<tr>
<td>Finnish</td>
<td>5149</td>
</tr>
<tr>
<td>Hungarian</td>
<td>5905</td>
</tr>
</tbody>
</table>

5 Math-Bridge: Technology-Enhanced Learning of Mathematics

The Math-Bridge platform provides students with multilingual, semantic and adaptive access to mathematical content. It has been developed based on the ActiveMath technology, and can be considered the next phase in the evolution of the ActiveMath intelligent tutoring system (Melis et al. 2001; Melis et al. 2006; Melis

² http://www.math-bridge.org/content/mathbridge.owl
³ http://wiki.math-bridge.org/display/ntns/Home
et al. 2009). Fig. 31.3 presents the dashboard of Math-Bridge. This is the entry point to the platform that every user sees after the login. Its interface consists of several widgets that provide access to different facilities available within Math-Bridge: regular and personalized courses, questionnaires, tests, and bookmarks.

![Math-Bridge dashboard](image)

**Fig. 31.3 Math-Bridge dashboard**

If a user clicks on any of the courses, the student interface of Math-Bridge will launch (Fig. 31.4). It consists of three panels. The left panel is used for navigation through learning material using the topic-based structure of the course. The topics can have subtopics and can be folded and unfolded.

When one of the bottom-level topics is chosen, the content page associated with this topic is presented in the central panel. Each page can consist of multiple LOs that a student can read. Exercise LOs have a button “Start Exercise”, which launches this particular interactive exercise in a separate tab. Students can also open new tabs when opening the results of the search and browsing through the LO metadata.

Whenever a LO is clicked on the content page, it gains focus. The right panel provides access to the details of the LO currently in focus, as well as some additional features, such as semantic search and social feedback toolbox. It can also be used for on-demand individual translation of the LO in focus to any language available for it, while the system interface and the rest of the content remain in the original language. This is one of the ways Math-Bridge supports international students. Alternatively, the entire system interface and course material can be accessed in any of the supported languages; the language can be specified at login.
Tracing Students’ Progress and Modeling Their Knowledge

Math-Bridge logs every student interaction with learning content. Actions, like loading a page or answering an exercise are stored in the student’s history database and help tracing and interpreting her/his learning progress.

The results of interactions with exercises (correct/incorrect/partially correct) are used by the student-modelling component of Math-Bridge to produce a meaningful estimation of the student’s progress. The student model of Math-Bridge supports multi-layered representation of student’s masteries. For every concept in the domain the model maintains a set of values estimating the probabilities that the student has mastered individual mathematical competencies associated with this concept. Every exercise in Math-Bridge is linked with one or several concepts (symbols, theorems, definitions etc.) and the competencies that the exercise is training for these concepts. A correct answer to the exercise is interpreted by the system as evidence that the student advances towards mastery and will result in the increase of probabilities for the corresponding concepts and competencies in this student’s model. (Faulhaber and Melis 2008) provides the architectural and implementation details of this student modelling mechanism.

Personalized Courses

The course generator component of Math-Bridge allows students to automatically assemble a course optimized for their needs and adapted to their knowledge and competencies based on the current states of their student models. To generate a course, students need to select the target topics and a learning scenario. Several scenarios are available within Math-Bridge: a student can choose to explore a new topic, train a particular competency, prepare for an exam, master a previous topic or assemble a course that will focus on the current gaps in student’s knowledge (Ullrich 2008; Biehler et al. 2012). Each course type is generated based on a set of pedagogical rules defining the top-level structure of the course and the learning goals. The generation tool queries the student model and the metadata storage in
order to assemble a didactically valid sequence of LOs. Pedagogical metadata (such as exercise difficulty) and semantic metadata (such as prerequisite-outcome relations) play the central role in this process. On the interface level, generation of a single course is a simple 4-step process (see Fig. 31.5). After that, the generated course appears in the “My Courses” widget, and the student can access it the same way s/he accesses standard pre-defined courses.

Adaptive Navigation Support
The amount of content available within Math-Bridge is massive. Some of the pre-defined bridging courses consist of thousands of LOs. Math-Bridge helps students finding the right page to read and/or the right exercise to attempt by implementing a popular adaptive navigation technique – adaptive annotation (Brusilovsky et al. 2009). The annotation icons show the student how much progress s/he has achieved for the corresponding part of learning material. Math-Bridge computes annotations on several levels: each course in the Math-Bridge dashboard (Fig. 31.3, widget “Courses”), each topic within a course table of contents (Fig. 31.4, left panel) and each content page under a topic are provided with individual progress indicators aggregating student’s learning activity on the corresponding level.
Interactive Exercises and Problem Solving Support

Interactive exercises play two important roles in Math-Bridge. First of all, they maintain constant assessment of students’ knowledge thus providing the input for the student-modelling component. Second, they give students the opportunity to train mathematical competencies and apply in practice theoretical knowledge acquired by reading the rest of the content.

The exercise subsystem of Math-Bridge can serve multi-step exercises with various types of interactive elements and rich diagnostic capabilities. At each step, Math-Bridge exercises can provide students with instructional feedback ranging from mere flagging the (in)correctness of given answers to presenting adaptive hints and explanations (the central panel of the student interface shown by Fig. 31.4 presents an example of feedback produced by a Math-Bridge exercise).

Math-Bridge can automatically generate interactive exercises powered by external domain reasoner services. Currently, Math-Bridge employs a collection of IDEAS domain reasoners for stepwise diagnosis of students’ actions and generation of advanced feedback on every step of their solutions (Heeren et al. 2010).

The Math-Bridge platform also implements functionality for integrating third-party exercise services that maintain the full cycle of student-exercise interaction. As a result, students can access within Math-Bridge both, native Math-Bridge exercises and exercises served by remote systems. The integration is seamless for the student (Math-Bridge makes no difference in how native and external exercises are launched) and fully functional (Math-Bridge makes no difference in how students’ interactions with native and external exercises are logged and interpreted by its modelling components). Currently Math-Bridge integrates two external exercise systems: STACK (Sangwin 2008) and mathe-online (Embacher 2006).

Semantic Search of LOs

In addition to navigating through the course topics, students have a more direct way to find LOs of their interest – by using the Math-Bridge search tool. They can use default search based on simple string matching, advanced search that allows more precise specification of general search parameters (exact or practical matching, lexical or phonetic matching) and semantic search. The semantic search mode fully utilizes the advanced metadata schema of Math-Bridge. Students can specify the type of the desired LO (e.g. only exercises), its difficulty (e.g. only easy exer-
Math-Bridge exercises), its target field of study (e.g. only easy exercises designed for physics students), etc. Fig. 31.6 presents the interface of the Math-Bridge search tool. The left part shows the results of simple search with the word “function”; the right part shows the results of semantic search with the same word and the target type of LOs restricted to assertions.

6 Technology-Enhanced Teaching of Mathematics

Math-Bridge offers teachers and university IT specialists a complete arsenal of tools necessary to setup, administer and teach online courses.

Content and Course management
Teachers can create their courses from scratch or reuse one of the existing tables of contents. They can design assessment tests, exams and questionnaires, and author individual LOs and collections of new material.

User and Group Management
There are three categories of users in the system. Students can access learning content individually or as a part of a course. Teachers can manage their courses, including content visibility and student roster. Administrators have access to all aspects of Math-Bridge user management. They can change user parameters and rights, modify group membership, and assign a teacher to a course. Naturally, administrators can also do everything that other users can.

Course Monitoring with the Reporting Tool
It is easy for teachers to monitor students’ progress within Math-Bridge: a dedicated reporting tool allows them to trace individual student’s performance or results of the entire class. The reporting tool can also help in discovering potentially problematic LOs (e.g. an exercise that nobody has solved correctly). Fig. 31.7 presents an outcome example of an aggregated report on the system usage. Overall, Math-Bridge provides teachers with about 10 different reports.
7 Conclusion

Math-Bridge is a full-fledged e-Learning platform developed to help individual learners, classes of students, as well as entire schools and universities to achieve their real-life educational goals. Math-Bridge implements a number of advanced technologies to support adaptive and semantic access to learning content. Fostering the adoption of these technologies by the general public is the primary goal of Math-Bridge. The platform has been evaluated in several experiments with more than 3000 students from nine universities and seven European countries. Detailed results of two of these experiments can be found in (Biehler et al. 2013) and (Kangas et al. 2012). Math-bridge has been used under different scenarios: as the main learning platform in a distant course, as an online component in a blended course, and as a supplementary tool in a traditional course. Under such diversity of usage scenarios and educational settings, it has been confirmed that students learn with Math-Bridge and that they in general feel very positive about using the tool. Further experiments are required to detect other effects of using Math-Bridge for learning and teaching remedial mathematics.

References


