A Proof Theoretic Interpretation of Model Theoretic Hiding

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Abstract. Logical frameworks like LF are used for formal representations of logics in order to make them amenable to formal machine-assisted meta-reasoning. While the focus has originally been on logics with a proof theoretic semantics, we have recently shown how to define model theoretic logics in LF as well. We have used this to define new institutions in the Heterogeneous Tool Set in a purely declarative way.
It is desirable to extend this model theoretic representation of logics to the level of structured specifications. Here a particular challenge among structured specification building operations is hiding, which restricts a specification to some export interface. Specification languages like ASL and CASL support hiding, using an institution-independent model theoretic semantics abstracting from the details of the underlying logical system.
Logical frameworks like LF have also been equipped with structuring languages. However, their proof theoretic nature leads them to a theory-level semantics without support for hiding. In the present work, we show how to resolve this difficulty.

1 Introduction

This work is about reconciling the model theoretic approach of algebraic specifications and institutions [AKKB99,ST11,GB92] with the proof theoretic approach of logical frameworks [HHP93,Pau94].

In [Rab10,CHK+10], we show how to represent institutions in logical frameworks, notably LF [HHP93], and extend the Heterogeneous Tool Set [MML07] with a mechanism to add new logics that are specified declaratively in a logical framework.

In the present work, we extend this to the level of structured specifications, including hiding. In particular, we will translate the ASL-style structured specifications with institutional semantics [SW83,Wir86,ST88] (also used in CASL [Mos04]) into the module system MMT [RK10] that has been developed in the logical frameworks community.

Like ASL, MMT is a generic structuring language that is parametric in the underlying language. But where ASL assumes a model theoretic base language – given as an institution – MMT assumes a proof theoretic base language given in terms of typing judgments. If we instantiate MMT with LF (as done in [RS09]),

we can represent both logics and theories as MMT-structured LF signatures. This is used in the LATIN project [KMR09] to obtain a large body of structured representations of logics and logic translations. An important practical benefit of MMT is that it is integrated with a scalable knowledge management infrastructure based on OMDoc [Koh06].

However, contrary to model theoretic structuring languages like ASL, structuring languages like MMT for logical frameworks have a proof theoretic semantics and do not support hiding, which makes them less expressive than ASL. Therefore, we proceed in two steps. Firstly, we extend LF+MMT with primitives that support hiding while preserving its proof theoretic flavor. Here we follow and extend the theory-level semantics for hiding given in [GR04]. Secondly, we assume an institution that has been represented in LF, and give a translation of ASL-structured specifications over it into the extended LF+MMT language.

The paper is organized as follows. In Sect. 2, we recall ASL- or CASL-style structured specifications with their institution-independent semantics; and in Sect. 3 we recall LF and MMT with its proof theoretic semantics. In Sect. 4, we extend MMT with hiding, and in Sect. 5, we define a translation of ASL style specifications into MMT and prove its correctness. Sect. 7 concludes the paper.

2 Structured specifications

Institutions [GB92] have been introduced as a formalization of the notion of logical systems. They abstract from the details of signatures, sentences and models and assume that signatures can be related via signature morphisms (which carries over to sentences and models).

Definition 1. *An* **institution** *is a quadruple* $I = (Sig, Sen, Mod, \models)$ *where:*

- Sig *is a category of* signatures*;*
- $Sen : Sig \to \mathcal{SET}$ *is a functor to the category* $\mathcal{SET}$ *of small sets and functions, giving for each signature* Σ *its set of* sentences $Sen(\Sigma)$ *and for any signature morphism* $\varphi : \Sigma \to \Sigma'$ *the* sentence translation *function* $Sen(\varphi) : Sen(\Sigma) \to Sen(\Sigma')$ *(denoted also* φ*);*
- $Mod : Sig^{op} \to Cat$ *is a functor to the category of categories and functors* Cat [1] *giving for any signature* Σ *its category of models* $Mod(\Sigma)$ *and for any signature morphism* $\varphi : \Sigma \to \Sigma'$ *the* model reduct *functor* $Mod(\varphi) : Mod(\Sigma') \to Mod(\Sigma)$ *(denoted* $\_|_\varphi$*);*
- *a satisfaction relation* $\models_\Sigma \subseteq |Mod(\Sigma)| \times Sen(\Sigma)$ *for each signature* Σ

such that the following satisfaction condition holds:

$$M'|_\varphi \models_{\Sigma'} e \Leftrightarrow M' \models_\Sigma \varphi(e)$$

for each $M' \in |Mod(\Sigma')|$ *and* $e \in Sen(\Sigma)$*, expressing that truth is invariant under change of notation and context.*

[1] We disregard here the foundational issues, but notice however that Cat is actually a so-called quasi-category.

For an institution I, a *presentation* is a pair (Σ, E) where Σ is a signature and E is a set of Σ-sentences. For a set E of Σ-sentences, $Mod_\Sigma(E)$ denotes the class of all Σ-models satisfying E. For a class $\mathcal{M}$ of Σ-models, $Th_\Sigma(\mathcal{M})$ denotes the set of all sentences that hold in every model in $\mathcal{M}$, and for a set of Σ-sentences E, we write $Cl_\Sigma(E)$ for $Th_\Sigma(Mod_\Sigma(E))$. Presentations provide the simplest form of specifications, and we refer to them as *basic specifications*.

Working with basic specifications is only suitable for specifications of fairly small size. For practical situations as in the case of large systems, they would become impossible to understand and use efficiently. Moreover, a modular design allows for reuse of specifications. Therefore, algebraic specification languages provide support for structuring specifications.

The semantics of (structured) specifications can be given as a signature and either (i) a class of models of that signature (*model-level semantics*) or (ii) a set of sentences over that signature (*theory-level semantics*). In the presence of structuring, the two semantics may be different in a sense that will be made precise below. The first algebraic specification language, Clear [BG80], used a theory-level semantics; the first algebraic specification language using model-level semantics for structured specifications was ASL [SW83,Wir86], whose structuring mechanisms were extended to an institution-independent level in [ST88].

In Fig. 1, we present a kernel of specification-building operations and their semantics over an arbitrary institution, similar to the one introduced in [ST88]. The second and third columns of the table contain the model-level and the theory-level semantics for the corresponding **structured specification** SP, denoted $Mod[SP]$ and $Thm[SP]$ respectively. The signature of SP, denoted $Sig[SP]$ is defined as follows: (i) $Sig[(\Sigma, E)] = \Sigma$, (ii) $Sig[SP_1 \cup SP_2] = Sig[SP_1]$ $(= Sig[SP_2])$, (iii) $Sig[\sigma(SP)] = \Sigma'$, where Σ' is the target signature of σ and finally (iv) $Sig[\sigma^{-1}(SP)] = \Sigma$, where Σ is the source signature of σ. Note that we restrict attention to hiding against inclusion morphisms. Moreover, we will only consider basic specifications that are finite.

SP	$Mod[SP]$	$Thm[SP]$
(Σ, E) $E \subseteq Sen(\Sigma)$	$Mod_\Sigma(E)$	$Cl_\Sigma(E)$
$SP_1 \cup SP_2$ $Sig[SP_1] = \Sigma$ $Sig[SP_2] = \Sigma$	$Mod(SP_1) \cap Mod(SP_2)$	$Cl_\Sigma(Thm[SP_1] \cup Thm[SP_2])$
$\sigma(SP)$ $\sigma : Sig[SP] \to \Sigma'$	$\{M \in Mod(\Sigma') \mid M\|_\sigma \in Mod[SP]\}$	$Cl_\Sigma(\{\sigma(e) \mid e \in Thm[SP]\})$
$\sigma^{-1}(SP)$ $\sigma : \Sigma \hookrightarrow Sig[SP]$	$\{M\|_\sigma \mid M \in Mod[SP]\}$	$\{e \in Sen(\Sigma) \mid \sigma(e) \in Thm[SP]\}$

Fig. 1. Semantics of Structured Specifications

Without hiding, the two semantics can be regarded as dual because we have $Thm[SP] = Th_{Sig[SP]}(Mod[SP])$, which is called *soundness* and *completeness* in [ST11]. But completeness does not hold in general in the presence of hiding [Bor02]. Moreover, in [ST11] it is proved that this choice for defining the theory level semantics is the strongest possible choice with good structural properties (e.g. compositionality). This shows that the mismatch between theory-level semantics and model-level semantics cannot be bridged in this way. We will argue below that this is not a failure of formalist methods in general; instead, we will pursue a different approach that takes model-level aspects into account while staying mechanizable.

The mismatch between model and theory-level semantics is particularly apparent when looking at **refinements**. For two Σ-specifications SP and SP', we write $SP \rightsquigarrow_\Sigma SP'$ if $Mod[SP'] \subseteq Mod[SP]$. Without hiding, this is equivalent to $Thm[SP] \subseteq Thm[SP']$, which can be seen as soundness and completeness properties for refinements. But in the presence of hiding, both soundness (if SP has hiding) and completeness (if SP' has hiding) for refinements may fail.

3 LF and MMT

The Edinburgh Logical Framework LF [HHP93] is a proof theoretic logical framework based on a dependent type theory related to Martin-Löf type theory [ML74]. Precisely, it is the corner of the λ-cube [Bar92] that extends simple type theory with dependent function types. We will also use the notion of LF signature morphisms as given in [HST94]. Moreover, in [RS09], LF was extended with a module system based on MMT. MMT [RK10] is a generic module system which structures signatures using named imports and signature morphisms. The expressivity of MMT is similar to that of ASL or development graphs [AHMS99] except for a lack of hiding. In [Rab10], LF is used as a logical framework to represent both proof and model theory of object logics.

We give a brief summary of basic LF signatures, MMT-structured LF signatures, and the representation of model theory in LF in Sect. 3.1, 3.2, and 3.3, respectively. Our approach is not restricted to LF and can be easily generalized to other frameworks such as Isabelle or Maude along the lines of [CHK+10].

3.1 LF

LF expressions E are grouped into kinds K, kinded type-families $A : K$, and typed terms $t : A$. The kinds are the base kind `type` and the dependent function kinds $\Pi x : A.\, K$. The type families are the constants a, applications $a\ t$, and the dependent function type $\Pi x : A.\, B$; type families of kind `type` are called types. The terms are constants c, applications $t\ t'$, and abstractions $\lambda x : A.\, t$. We write $A \to B$ instead of $\Pi x : A.\, B$ if x does not occur in B. An LF **signature** Σ is a list of kinded type family declarations $a : K$ and typed constant declarations $c : A$. Optionally, declarations may carry definitions. A grammar that subsumes LF is given in Sect. 3.2 below.

Given two signatures Σ and Σ', an LF **signature morphism** $\sigma : \Sigma \to \Sigma'$ is a typing- and kinding-preserving map of Σ-symbols to Σ'-expressions. Thus, σ maps every constant $c : A$ of Σ to a term $\sigma(c) : \overline{\sigma}(A)$ and every type family symbol $a : K$ to a type family $\sigma(a) : \overline{\sigma}(K)$. Here, $\overline{\sigma}$ is the homomorphic extension of σ to Σ-expressions, and we will write σ instead of $\overline{\sigma}$ from now on. Signature morphisms preserve typing and kinding: if $\vdash_\Sigma E : E'$, then $\vdash_{\Sigma'} \sigma(E) : \sigma(E')$.

Composition and identity of signature morphisms are straightforward, and we obtain a category $\mathbb{LF}$ of LF signatures and morphisms. This category has **inclusion** morphisms by taking inclusions between sets of declarations. Moreover, it has **pushouts** along inclusions [HST94]. Finally, a **partial morphism** from Σ to Σ' is a signature morphism from a subsignature of Σ to Σ'. Partiality will only be used in MMT structures below, and we do not need to define a composition of partial morphisms.

LF uses the Curry-Howard correspondence to represent axioms as constants and theorem as defined constants (whose definiens is the proof). Then the typing-preservation of signature morphisms corresponds to the theorem preservation of theory morphisms.

3.2 LF+MMT

The motivation behind the MMT structuring operations is to give a flattenable, concrete syntax for a module system on top of a declarative language. Signature morphisms are used as the main concept to relate and form modular signatures, and signature morphisms can themselves be given in a structured way. Moreover, signature morphisms are always named and can be composed into morphism expressions.

The grammar for the LF+MMT language is given below where $[-]$ denotes optional parts. Object level expressions E unify LF terms, type families, and kinds, and morphism level expressions are composed morphisms:

Signature graph	$G ::= \cdot \mid G,\ \%\texttt{sig}\, T = \{\Sigma\} \mid \%\texttt{view}\, v : S \to T = \{\sigma\}$
Signatures	$\Sigma ::= \cdot \mid \Sigma,\ \%\texttt{struct}\, s : S = \{\sigma\} \mid \Sigma,\ c : E[= E']$
Morphisms	$\sigma ::= \cdot \mid \sigma,\ \%\texttt{struct}\, s := \mu \mid \sigma,\ c := E$
Object level expr.	$E ::= \texttt{type} \mid c \mid x \mid E\ E \mid \lambda x : E.\, E \mid \Pi x : E.\, E \mid E \to E$
Morphism level expr.	$\mu ::= \cdot \mid T.s \mid v \mid \mu\ \mu'$

The $\mathbb{LF}$ signatures and signature morphisms are those without the keyword $\%\texttt{struct}$. Those are called **flat**[2].

Syntax. The module level declarations consist of named signatures R, S, T and two kinds of signature morphism declarations. Firstly, **views** $\%\texttt{view}\, v : S \to T = \{\sigma\}$ occur on toplevel and declare an explicit morphism from S to T given by σ. Secondly, **structures** $\%\texttt{struct}\, s : S = \{\sigma\}$ occur in the body of a signature T and declare an import from S into T. Structures carry a partial morphism σ from S to T, i.e., σ maps some symbols of S to expressions over T.

[2] Note that in the grammars presented in this paper $\cdot$ stands for the empty entity.

Views and structures correspond to refinements and inclusion of subspecifications in unions in ASL and CASL.

MMT differs from ASL-like structuring languages in that it uses named imports. Consequently, the syntax of MMT can refer to all paths in the signature graph using composed morphisms; these morphism level expressions μ are formed from structure names $T.s$, view names v, and diagram-order composition $\mu\,\mu'$.

MMT considers morphisms μ from S to T as expressions on the module level. Such a morphism μ has type S and is valid over T. Most importantly, MMT permits structured morphisms: The morphisms σ occurring in views and structures from S to T may map a structure $\%\texttt{struct}\,r : R = \{\sigma\}$ declared in S (i.e., a morphism level constant of type R over S) to a morphism $\mu : R \to T$ (i.e., a morphism level expression of type R over T). These are called structure maps $\%\texttt{struct}\,r := \mu$.

Semantics. The semantics of LF+MMT is given by **flattening**. Every well-formed LF+MMT signature graph G is flattened into a diagram $\overline{G}$ over $\mathbb{LF}$. Every signature S in G produces a node $\overline{S}$ in $\overline{G}$; every structure $\%\texttt{struct}\,s : S = \{\sigma\}$ occurring in T produces an edge $\overline{T.s}$ from $\overline{S}$ to $\overline{T}$; and every view $\%\texttt{view}\,v : S \to T = \{\sigma\}$ produces an edge $\overline{v}$ from $\overline{S}$ to $\overline{T}$. Accordingly, every morphism expression μ yields a morphism $\overline{\mu}$. These results can be found in [RS09], and we will only sketch the central aspects here.

The flattening is defined by recursively replacing all structure declarations and structure maps with lists of flat declarations. To flatten a structure declaration $\%\texttt{struct}\,s : S = \{\sigma\}$ in a signature T, assume that S and σ have been flattened already. For every declaration $c : E[= E']$ in $\overline{S}$, we have in $\overline{T}$

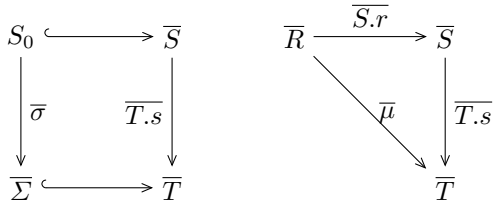
- a declaration $s.c : \overline{T.s}(E) = E''$ in $\overline{S}$ if σ contains $c := E''$,
- a declaration $s.c : \overline{T.s}(E)\,[= \overline{T.s}(E')]$ in $\overline{S}$ otherwise.

The morphism $\overline{T.s}$ from $\overline{S}$ to $\overline{T}$ maps every $\overline{S}$-symbol c to the $\overline{T}$ symbol $s.c$.

For a view $\%\texttt{view}\,v : S \to T = \{\sigma\}$, the morphism $\overline{v}$ from $\overline{S}$ to $\overline{T}$ is given by the flattening of σ. $\bar{\cdot}$ is the identity morphism in $\mathbb{LF}$, and $\overline{\mu\,\mu'}$ is the composition $\overline{\mu'} \circ \overline{\mu}$.

Finally, morphisms σ from S to T are flattened as follows. To flatten a structure map $\%\texttt{struct}\,r := \mu$ where r is a structure from R to S, assume that R has been flattened already. Then the flattening of σ contains $s.c := \overline{\mu}(c)$ for every constant c in $\overline{R}$.

In particular, if $\%\texttt{sig}\,T = \{\Sigma,\ \%\texttt{struct}\,s : S = \{\sigma\}\}$ the semantics of signature graphs is such that the left diagram below is a pushout. Here S_0 is a subsignature of $\overline{S}$ such that $\overline{\sigma} : S_0 \to \overline{\Sigma}$. Moreover, if S declares a structure r of type R, then the semantics of a structure map $\%\texttt{struct}\,r := \mu$ occurring in σ is that the diagram on the right commutes.

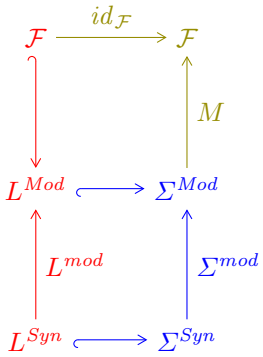


3.3 Representing Logics in LF

LF has been designed for the representation of the proof theory of logics. Recently we showed how this can be extended to representations of the model theory [Rab10]. The key idea is to use a signature Σ^{Mod} that represents models of a logical signature Σ and a signature $\mathcal{F}$ that represents the foundation of mathematics. Then individual Σ-models can be represented as morphisms $\Sigma^{Mod} \to \mathcal{F}$.

The feasibility of this approach has been demonstrated in [IR11], where we give detailed encodings of ZFC set theory, Mizar's set theory, and Isabelle's higher-order logic. Thus, we can choose the right foundation for every individual logic. In the following, it is sufficient to assume a fixed arbitrary signature $\mathcal{F}$. A comprehensive example has been given in [HR11] where we represent first-order logic with a set theoretical foundation. We will use a simplified variant of this methodology in the sequel and give a summary below, using first-order logic as a running example.

The commuting LF diagram on the right presents the representation of a logic L as a tuple $(L^{Syn}, L^{Mod}, L^{mod}, \mathcal{F})$.



L^{Syn} represents the syntax of the logic. We assume that L^{Syn} contains two distinguished declarations o : `type` and `ded` : $o \to$ `type`. For example, for first-order logic, L^{Syn} contains a constant $\wedge : o \to o \to o$ for conjunction along with proof rules for it. Then Σ-sentences are represented as closed $\beta\eta$-normal terms of type o over Σ^{Syn}, and correspondingly Σ-proofs of F as terms of type `ded` F. Theorems are represented as sentences F for which the type `ded` F is inhabited over Σ^{Syn}.

$\mathcal{F}$ represents the foundation of mathematics. In the case of set theory, $\mathcal{F}$ contains in particular symbols *set* : `type`, *prop* : `type` and *true* : *prop* $\to$ `type` for sets, propositions, and proofs. Moreover, it contains predicates $\in$: *set* $\to$ *set* $\to$ *prop* and *eq* : *set* $\to$ *set* $\to$ *prop* for elementhood and equality between sets.

L^{Mod} extends $\mathcal{F}$ with declarations for all components present in any model. For example, to represent set theoretical first-order models, L^{Mod} declares two symbols: a set *univ* : *set* for the universe and an axiom making *univ* non-empty.

L^{mod} interprets the syntax in a model. It represents the fixed part of the interpretation function assigning semantics to the logical symbols. For example, in the case where $\mathcal{F}$ is set theory, $L^{mod}(o)$ could be an $\mathcal{F}$-type representing the set $\{0,1\}$ of booleans, and $L^{mod}(\texttt{ded})$ the type family $\lambda x.\, true\ (x\ eq\ 1)$. Then L^{mod} would map $\wedge$ to the $\mathcal{F}$-expression representing the boolean AND function.

Individual signatures Σ are represented as inclusion morphisms $L^{syn} \hookrightarrow \Sigma^{Syn}$. In the case of first-order logic, Σ^{Syn} extends L^{Syn} with declarations for all the function and predicate symbols. Due to the Curry-Howard representation of proofs as terms, there is no conceptual difference between representing signatures and theories of the underlying logic. Therefore, Σ^{Syn} could also declare axioms.

From these, we obtain Σ^{Mod} and Σ^{mod} via a pushout in the category of LF signatures. Thus, Σ^{Mod} arises as the extension of L^{Mod} with declarations for all components present in a model of Σ. In our running example, Σ^{Mod} would declare, e.g., an n-ary function/relation on $univ$ for every n-ary function/predicate symbol declared in Σ^{Syn}.

Then a model M of Σ can finally be represented as a morphism $M : \Sigma^{Mod} \to \mathcal{F}$ such that $M|_{\mathcal{F}} = id_{\mathcal{F}}$. In our running example, $M(univ)$ is the universe of the model, and M maps every non-logical symbol declared in Σ^{Mod} to its interpretation.

Then finally, for a sentence $F : o$ over Σ^{Syn}, the homomorphic extension $M(\Sigma^{mod}(F))$ yields the truth value of F in M. In particular, the satisfaction of F in M is represented as the inhabitation of the type $M(\Sigma^{mod}(\texttt{ded}\ F))$ over $\mathcal{F}$. If Σ^{Syn} contains axioms, then so does Σ^{Mod}. In that case, M must map each axiom to a proof in $\mathcal{F}$. Thus, the type-preservation property of LF signature morphisms guarantees that all such morphisms indeed yield models.

In [Rab10], the proof theory of the logic is represented in parallel to the model theory as a morphism $L^{pf} : L^{Syn} \to L^{Pf}$ where L^{Pf} adds the proof rules that populate the types $\texttt{ded}\, F$. Here, we assume for simplicity that $L^{Syn} = L^{Pf}$, and our results easily extend to the general case.

4 Hiding in LF and MMT

In a proof theoretic setting as in LF+MMT, flattening is not a theorem but rather the way to assign meaning to a modular signature. Since hiding precludes flattening, it is a particularly difficult operation to add to systems like LF+MMT.

In this section, we develop an extension of LF+MMT with hiding. The basic idea is to represent signatures with hidden information as inclusions $\Sigma_v \hookrightarrow \Sigma_h$. Intuitively, Σ_v contains all declarations making up the visible interface, and $\Sigma_h \setminus \Sigma_v$ contains declarations for all the hidden operations. Thus, the hidden operations are never removed; instead, they are recorded in Σ_h. That way the hidden operations are still available when defining models. Intuitively, models will be represented as morphisms out of Σ_v that factor through Σ_h.

In the following, we will abstractly introduce LF signatures with hidden declarations and morphisms between such signatures in Sect. 4.1. They will be plain

LF-signatures with a distinguished subsignature for the visible declarations. Intuitively, an LF signature with hiding is a plain LF-signature in which some declarations are flagged as hidden. These LF signatures with hiding do not use the module system yet, and we will extend the MMT module system to LF signatures with hiding in Sect. 4.2. Intuitively, LF+MMT with hiding works in exactly the same way as LF+MMT except for keeping track of which declarations are hidden.

4.1 LF with Hiding

We will not only introduce LF signatures with hidden declarations but also LF morphisms that hide constants. It is important to realize that we need hiding morphisms in addition to partial morphisms. Therefore, we introduce H-morphisms, which may have the two orthogonal properties of total/partial and revealing/hiding. Here *revealing* is used for H-morphisms that do not use hiding: The (partial) revealing H-morphisms will be exactly the (partial) $\mathbb{LF}$-morphisms from above.

Given two $\mathbb{LF}$-signatures Σ and Σ', an **H-morphism** from Σ to Σ' consists of two subsignatures $\Sigma_0 \hookrightarrow \Sigma_1 \hookrightarrow \Sigma$ and an $\mathbb{LF}$ signature morphism $\sigma : \Sigma_0 \to \Sigma'$. The intuition is that σ maps all constants in Σ_0 to Σ'-expressions and hides all constants in $\Sigma \setminus \Sigma_1$; for the intermediate declarations in $\Sigma_1 \setminus \Sigma_0$, σ is left undefined, i.e., partial. We call Σ_0 the **revealed domain** and Σ_1 the **non-hidden domain** of σ. We call σ **total** if $\Sigma_1 = \Sigma_0$ and otherwise **partial**; and we call σ **revealing** if $\Sigma = \Sigma_1$ and otherwise **hiding**.

For a Σ-expression E, we say that σ **maps** E if E is a Σ_0-expression and that σ **hides** E if E is not a Σ_1-expression. Then we can define a **composition** of total H-morphisms as follows: The revealed domain of $\sigma' \circ \sigma$ is the largest subsignature of the revealed domain of σ that comprises only constants c such that σ' maps $\sigma(c)$; then we can put $(\sigma' \circ \sigma)(c) = \sigma'(\sigma(c))$. We omit the technical proof that the revealed domain of $\sigma' \circ \sigma$ well-defined.

An **H-signature** is a pair $\Sigma = (\Sigma_v, \Sigma_h)$ such that Σ_v is a subsignature of Σ_h. We call Σ_h the **domain** and Σ_v the **visible domain** of Σ.

Finally, we define the category $\mathbb{LFH}$ whose objects are H-signatures and whose morphisms $(\Sigma_v, \Sigma_h) \to (\Sigma'_v, \Sigma'_h)$ are total H-morphisms from Σ_v to Σ'_v. Note that these morphisms are exactly the total morphisms from Σ_h to Σ'_v whose revealed domain is at most Σ_v. The $\mathbb{LFH}$ identity of (Σ_v, Σ_h) is the $\mathbb{LF}$ identity of Σ_v. Associativity follows after observing that $\sigma'' \circ (\sigma' \circ \sigma)$ hides c iff $(\sigma'' \circ \sigma') \circ \sigma$ hides c.

$\mathbb{LFH}$-morphisms only translate between the visible domains and may even use hiding in doing so. We are often interested in whether the hidden information could also be translated. Therefore, we define:

Definition 2. *For an $\mathbb{LFH}$-morphism $\sigma_0 : \Sigma \to \Sigma'$ with revealed domain Σ_0, we write $\sigma_0 : \Sigma \xrightarrow{!} \Sigma'$ if σ_0 can be extended to a total revealing morphism $\sigma : \Sigma_h \to \Sigma'_h$, i.e., if there is an $\mathbb{LF}$ morphism $\sigma : \Sigma_h \to \Sigma'_h$ that agrees with σ_0 on Σ_0.*

4.2 LF+MMT with Hiding

We can now extend the MMT structuring to $\mathbb{LFH}$, i.e., to a base language with hiding. The flattening of signature graphs with hiding will produce $\mathbb{LFH}$-diagrams.

We avoid using pairs (Σ_v, Σ_h) in the concrete syntax for H-signatures and instead extend the grammar of LF+MMT as follows:

$$\begin{array}{l}\text{Signatures } \Sigma ::= \cdot \mid \Sigma,\ [\%\texttt{hide}]\ \%\texttt{struct}\, s : S = \{\sigma\} \mid \Sigma,\ [\%\texttt{hide}]\ c : E[= E] \\ \text{Morphisms } \sigma ::= \cdot \mid \sigma, \%\texttt{struct}\, s := \mu \mid \sigma, c := E \mid \%\texttt{hide}\, c \mid \%\texttt{hide}\, \%\texttt{struct}\, s\end{array}$$

If a declaration in Σ has the $\%\texttt{hide}$ modifier, we call it **hidden**, otherwise **visible**. Hidden declarations are necessary to keep track of the hidden information. From a proof theoretical perspective, it may appear more natural to delete them, but this would not be adequate to represent ASL specifications with hiding.

If σ contains $c := E$ (or $\%\texttt{hide}\, c$), we say that σ **maps** (or **hides**) c, and accordingly for structures. As before, we call signatures or morphisms **flat** if they do not contain the $\%\texttt{struct}$ keyword.

The semantics of a well-formed signature graph G is given in two steps: first G is flattened into a flat signature graph $\widetilde{G}$, second the semantics of a flat signature graph G is given by an $\mathbb{LFH}$-diagram $\overline{G}$. In particular, every composite μ from S to T occurring in G induces a total H-morphism $\overline{\mu} : \overline{S}_v \to \overline{T}_v$.

Well-formedness and semantics are defined in a joint induction on the structure of G, and only minor adjustments to the definition of $\overline{G}$ for LF+MMT are needed. We begin with the flat syntax.

Firstly, a flat signature $\%\texttt{sig}\, T = \{\Sigma\}$ induces a hiding signature $\overline{T} = (\overline{T}_v, \overline{T}_h)$ as follows: $\overline{T}_h$ contains all declarations in Σ, and $\overline{T}_v$ is the largest subsignature of $\overline{T}_h$ that contains only visible declarations. Σ is well-formed if this is indeed a well-formed $\mathbb{LFH}$-object.

Secondly, consider a flat morphism σ and two flat signatures S and T in G. σ induces an H-morphism from $\overline{S}_v$ to $\overline{T}_v$ as follows: Its revealed domain is the smallest subsignature of $\overline{S}_v$ that contains all constants mapped by σ; its non-hidden domain is the largest subsignature of $\overline{S}_v$ that contains no constants hidden by σ. σ is well-formed if this is indeed a well-formed H-morphism from $\overline{S}_v$ to $\overline{T}_v$.

Next we define the semantics of the full syntax by flattening an arbitrary signature graph G to $\widetilde{G}$. We use the same definition as in [RS09] except for additionally keeping track of which declarations are hidden. In particular, the semantics is unchanged if no declarations are hidden.

Firstly, consider a signature T with a structure $\%\texttt{struct}\, s : S = \{\sigma\}$, and consider a declaration of c in $\widetilde{S}$. Then $\widetilde{T}$ contains a constant $s.c$ defined in the same way as for LF+MMT. Moreover, $s.c$ is hidden in $\widetilde{T}$ if s is hidden in T, c is hidden in S, or $\widetilde{\sigma}$ hides c.

Secondly, consider an occurrence of $\%\texttt{struct}\, s := \mu$ in σ in a structure or view declaration with domain S. Since the semantics $\overline{\mu}$ of μ is a total H-morphism, we

must consider two cases for every visible constant c in $\widetilde{S}$: if c is in the revealed domain of $\overline{\mu}$, then $\widetilde{\sigma}$ contains $s.c := \overline{\mu}(c)$ as for LF+MMT; otherwise, $\widetilde{\sigma}$ contains $\%\texttt{hide}\, s.c$.

Thirdly, consider an occurrence of $\%\texttt{hide}\,\%\texttt{struct}\, s$ in σ in a structure or view declaration with domain S. Then $\widetilde{\sigma}$ contains $\%\texttt{hide}\, s.c$ for every visible constant c of $\widetilde{S}$.

Finally, to define well-formedness of signature graphs, we use the same inference system as in [RS09] with the following straightforward restriction for morphisms: In a structure declaration $\%\texttt{struct}\, s : S = \{\sigma\}$ within T or in a view declaration $\%\texttt{view}\, v : S \to T = \{\sigma\}$, $\overline{\widetilde{\sigma}}$ must be an H-morphism from $\overline{\widetilde{S}}_v$ to $\overline{\widetilde{T}}_v$. $\overline{\widetilde{\sigma}}$ must be total for views and may be partial for structures. Such structures and views induce edges $\overline{T.s}$ and $\overline{v}$ in $\overline{G}$ in the obvious way.

It is easy to show that well-formedness of the flat syntax is decidable. Moreover, the following result can be proved by a straightforward induction on the structure of G.

Theorem 3. *$\overline{\widetilde{G}}$ is a diagram over $\mathbb{LFH}$ for every well-formed signature graph G.*

The morphisms σ in structures and views may only map symbols of the visible domain. Moreover, they may hide some of these symbols. However, if we inspect the definition of the flattening of a structure $\%\texttt{struct}\, s : S = \{\sigma\}$, we see that it imports all constants of $\overline{S}$ including the hidden ones and including those hidden by σ. Therefore, we have:

Lemma 4. *Assume a well-formed signature graph with hiding G containing a structure $\%\texttt{struct}\, s : S = \{\sigma\}$ in T. Then $\overline{T.s} : \overline{S} \xrightarrow{!} \overline{T}$.*

Proof. The extension of $\overline{T.s}$ to $\overline{S}_h$ maps every constant c to $s.c$.

5 Interpreting ASL in LF+MMT

We now introduce the translation from ASL-style structured specifications into LF+MMT. We assume that there is a representation of an institution I in LF (see Sect. 5.1), such that when translating an ASL-style specification over I (see Sect. 5.2), the resulting MMT specification is based on this representation. The subsequent subsections deal with proving adequacy of the translation.

5.1 Logics

Consider an encoding as in Sect. 3 for an institution I. We make the following assumptions about the adequacy of the encoding.

Definition 5. *We say that a* **foundation** *is* **adequately** *represented by an LF signature $\mathcal{F}$ if there is (i) an $\mathcal{F}$-type prop* : `type` *such that there is a bijective*

representation $\ulcorner - \urcorner$ *of formal statements* F *of the foundation as* $\beta\eta$*-normal* $\mathcal{F}$*-terms* $\ulcorner F \urcorner : prop$*, and (ii) an* $\mathcal{F}$*-type family* $true : prop \to \texttt{type}$ *such that there is a bijective representation of formal proofs of* F *in the foundation as* $\beta\eta$*-normal* $\mathcal{F}$*-terms of type* $true\ \ulcorner F \urcorner$*.*

In the following, we will assume a fixed signature $\mathcal{F}$ that adequately represents the foundation of mathematics, in which the models of our specifications are expressed. For example, in order to represent an institution whose models are defined in terms of Zermelo-Fraenkel set theory, $\mathcal{F}$ can be the signature given in [HR11]; in that case the terms of type *prop* represent first-order formulas over the binary predicate symbol $\in$.

Definition 6. *We say that an* **institution** I *of the form* $(Sig, Sen, Mod, \models)$ *is* **adequately** *represented as* $(L^{Syn}, \mathcal{F}, L^{Mod}, L^{mod})$ *if there is a functor* $\Phi : Sig \to \mathbb{LF}/L^{Syn}$ *such that for every signature* Σ *(i)* $\Phi(\Sigma) = \Sigma^{Syn}$ *is an extension of* L^{Syn}*, (ii) there is a bijection* $\ulcorner - \urcorner$ *mapping* Σ*-sentences to* $\beta\eta$*-normal* Σ^{Syn}*-terms of type* o*, and* $\ulcorner - \urcorner$ *is natural with respect to sentence translation* $Sen(\sigma)$ *and morphism application* $\Phi(\sigma)$ *(iii) there is a bijection* $\ulcorner - \urcorner$ *mapping* Σ*-models to* $\mathbb{LF}$*-morphisms* $\Sigma^{Mod} \to \mathcal{F}$*, and* $\ulcorner - \urcorner$ *is natural with respect to model reduction* $Mod(\sigma)$ *and precomposition with* $\Phi(\Sigma)^{mod}$*, (iv) satisfaction* $M \models_\Sigma F$ *holds iff* $\ulcorner M \urcorner$ *maps* $\Sigma^{mod}(\ulcorner F \urcorner)$ *to an inhabited* $\mathcal{F}$*-type.*

Using the definitions of [Rab10], this can be stated more concisely as an institution comorphism from I to an appropriate institution based on LF.

Our assumption of a bijection between I-models and $\mathbb{LF}$-morphisms is quite strong. In most cases, not all models will be representable as morphisms. However, using canonical models constructed in completeness proofs, in many cases it will be possible to represent all models up to elementary equivalence.

Moreover, note that the bijection between models also directly implies that the represented institution has amalgamation, provided that we are able to transfer pushouts into the representation:

Theorem 7. *If the functor* Φ *of an adequate representation of an institution preserves pushouts, the represented institution* I *has amalgamation.*

5.2 Specifications

We define a translation from ASL specifications to LF+MMT signatures with hiding. Since the ASL structuring is built over an arbitrary institution, we assume that the underlying institution has already been represented in LF and the representation is adequate in the sense of Sect. 3.3 and Def. 6.

For every ASL specification SP over a signature Σ, we define two LF+MMT signatures of the form

$$\%\texttt{sig}\, N_\Sigma = \{\%\texttt{struct}\, l : L^{Syn}, \ulcorner \Sigma \urcorner\} \quad \%\texttt{sig}\, N_{SP} = \{\%\texttt{struct}\, s : N_\Sigma, \ulcorner SP \urcorner\}$$

$\ulcorner \Sigma \urcorner$ is a list of declarations representing the visible signature symbols of SP, and similarly $\ulcorner SP \urcorner$ represents the hidden signature symbols and all the axioms.

There is some flexibility regarding the treatment of axioms: Alternatively, we could distinguish visible and hidden axioms and put the visible ones in $\ulcorner\Sigma\urcorner$. Our choice makes the technical details simpler.

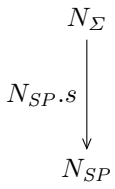
$\ulcorner\Sigma\urcorner$ and $\ulcorner SP\urcorner$ need to refer to the logical symbols of the underlying logic. Therefore, N_Σ starts with an import from L^{Syn}.

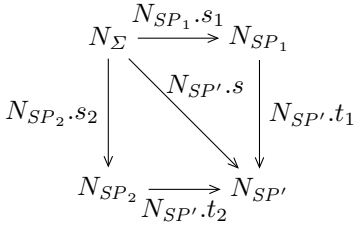
It is important to realize that LF+MMT does not use signature expressions in the way ASL uses specification-building operations. In LF+MMT, imports are achieved by declaring structures within the body of a signature, and for efficiency reasons, they may import only named signatures. Therefore, the translation from ASL to LF+MMT must translate nested specification-building operations into multiple named LF+MMT signatures; consequently, it results in an LF+MMT signature graph. We will use N_Σ and N_{SP} to denote the fresh names generated during the translation for the nodes and edges of this signature graph. Note that this leads to an increase in size but not to the exponential blow-up incurred when flattening.

In the following, we will define $\ulcorner\Sigma\urcorner$ and $\ulcorner SP\urcorner$ inductively on the structure of the specification SP. The cases of the translation are given in Fig. 2.

We will describe the translation step by step visualizing the involved objects using diagrams in $\mathbb{LF}$. First we introduce one simplification of the notation. Recall that technically, the semantics $\overline{N_{SP}}$ of N_{SP} is an $\mathbb{LFH}$ object $(\overline{N_{SP}}_v, \overline{N_{SP}}_h)$ and similarly for $\overline{N_\Sigma} = (\overline{N_\Sigma}_v, \overline{N_\Sigma}_h)$. A simple induction will show that N_Σ never contains hiding and that $\overline{N_{SP}.s} : \overline{N_\Sigma}_v = \overline{N_\Sigma}_h \to \overline{N_{SP}}_v$ is an isomorphism in $\mathbb{LF}$. Therefore, we will always write N_Σ instead of $\overline{N_\Sigma}_v$, N_{SP} instead of $\overline{N_{SP}}_h$, and $N_{SP}.s$ instead of $\overline{N_{SP}.s}$.

The rule *Basic* translates basic specification $SP = (\Sigma, E)$ using the LF representation of the underlying institution. $\ulcorner\Sigma\urcorner$ contains one declaration for every non-logical symbol declared in Σ. For example, if L^{Syn} encodes first-order logic and has a declaration $i : \texttt{type}$ for the universe, a binary predicate symbol p in Σ leads to a declaration $p : l.i \to l.i \to l.o$ in $\ulcorner\Sigma\urcorner$. All axioms $F \in E$, lead to a declaration $a : \texttt{ded}\ulcorner F\urcorner$ where a is a fresh name. This has the effect that axioms are always hidden, which simplifies the notation significantly; it is not harmful because the semantics of ASL does not depend on whether an axiom is hidden or visible.





The rule *Union* assumes translations of Σ, SP_1, and SP_2 and creates the translation of $SP' = SP_1 \cup SP_2$ by instantiating N_{SP_1} and N_{SP_2} in such a way that they share N_Σ. The semantics of LF+MMT guarantees that the resulting diagram on the left is a pushout in $\mathbb{LF}$.

The rule *Transl* translates $SP' = \sigma(SP)$ assuming that σ and SP have been translated already. The signature morphism σ is translated to a view in a straightforward way. Recall that N_Σ and $N_{\Sigma'}$ contain no hidden

$$\frac{SP := (\Sigma, \{F_1, \ldots, F_n\})}{\begin{array}{l}\%\texttt{sig}\, N_\Sigma \;=\; \{\%\texttt{struct}\, l : L^{Syn}, \ulcorner \Sigma \urcorner\} \\ \%\texttt{sig}\, N_{SP} \;=\; \{\%\texttt{struct}\, s : N_\Sigma, \\ \qquad \%\texttt{hide}\, a_1 : \texttt{ded}\, \ulcorner F_1 \urcorner, \ldots, \%\texttt{hide}\, a_n : \texttt{ded}\, \ulcorner F_n \urcorner\}\end{array}}\ \textit{Basic}$$

$$\frac{\begin{array}{ll}\Sigma = Sig[SP_1] = Sig[SP_2] & \%\texttt{sig}\, N_\Sigma \;=\; \{\%\texttt{struct}\, l : L^{Syn}, \ulcorner \Sigma \urcorner\} \\ SP' := SP_1 \cup SP_2 & \%\texttt{sig}\, N_{SP_1} \;=\; \{\%\texttt{struct}\, s_1 : N_\Sigma, \ulcorner SP_1 \urcorner\} \\ & \%\texttt{sig}\, N_{SP_2} \;=\; \{\%\texttt{struct}\, s_2 : N_\Sigma, \ulcorner SP_2 \urcorner\}\end{array}}{\begin{array}{l}\%\texttt{sig}\, N_{SP'} \;=\; \{\%\texttt{struct}\, s : N_\Sigma, \\ \qquad \%\texttt{struct}\, t_1 : N_{SP_1} \;=\; \{\%\texttt{struct}\, s_1 := s\}, \\ \qquad \%\texttt{struct}\, t_2 : N_{SP_2} \;=\; \{\%\texttt{struct}\, s_2 := s\}\}\end{array}}\ \textit{Union}$$

$$\frac{\begin{array}{ll}\sigma : \Sigma \to \Sigma' & \%\texttt{sig}\, N_\Sigma \;=\; \{\%\texttt{struct}\, l : L^{Syn}, \ulcorner \Sigma \urcorner\} \\ SP' := \sigma(SP) & \%\texttt{sig}\, N_{\Sigma'} \;=\; \{\%\texttt{struct}\, l' : L^{Syn}, \ulcorner \Sigma' \urcorner\} \\ & \%\texttt{sig}\, N_{SP} \;=\; \{\%\texttt{struct}\, s : N_\Sigma, \ulcorner SP \urcorner\} \\ & \%\texttt{view}\, N_\sigma : N_\Sigma \to N_{\Sigma'} = \{\%\texttt{struct}\, l := l', \ulcorner \sigma \urcorner\}\end{array}}{\begin{array}{l}\%\texttt{sig}\, N_{SP'} \;=\; \{\%\texttt{struct}\, s' : N_{\Sigma'}, \\ \qquad \%\texttt{struct}\, t : N_{SP} \;=\; \{\%\texttt{struct}\, s := N_\sigma s'\}\}\end{array}}\ \textit{Transl}$$

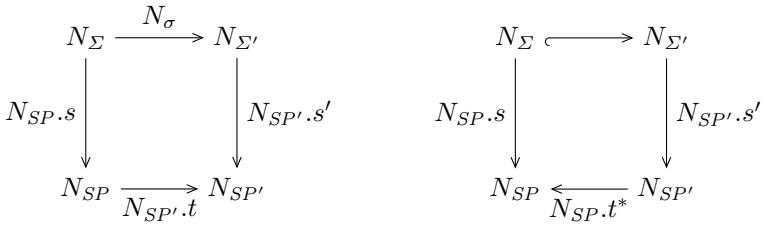
$$\frac{\begin{array}{ll}\sigma : \Sigma \hookrightarrow \Sigma' & \%\texttt{sig}\, N_\Sigma \;=\; \{\%\texttt{struct}\, l : L^{Syn}, \ulcorner \Sigma \urcorner\} \\ dom(\Sigma) = \{c_1, \ldots, c_m\} & \%\texttt{sig}\, N_{\Sigma'} \;=\; \{\%\texttt{struct}\, l' : L^{Syn}, \ulcorner \Sigma' \urcorner\} \\ dom(\Sigma') \setminus dom(\Sigma) = \{h_1, \ldots, h_n\} & \%\texttt{sig}\, N_{SP'} \;=\; \{\%\texttt{struct}\, s' : N_{\Sigma'}, \ulcorner SP' \urcorner\} \\ SP := \sigma^{-1}(SP') & \end{array}}{\begin{array}{l}\%\texttt{sig}\, N_{SP} \;=\; \{\%\texttt{struct}\, s : N_\Sigma, \\ \qquad \%\texttt{struct}\, t : N_{SP'} \;=\; \{\%\texttt{struct}\, s'.l' := s.l, \\ \qquad\qquad s'.c_1 := s.c_1, \ldots, s'.c_m := s.c_m, \\ \qquad\qquad \%\texttt{hide}\, s'.h_1, \ldots, \%\texttt{hide}\, s'.h_n\}\}\end{array}}\ \textit{Hide}$$

Fig. 2. Translation of ASL specifications to LF+MMT with Hiding

declarations or axioms so that N_σ is a (total) morphism in $\mathbb{LF}$. The resulting diagram is the left diagram below; it is again a pushout in $\mathbb{LF}$.

Similarly, the rule *Hide* translates $SP = \sigma^{-1}(SP')$ assuming that SP has been translated already. As $\sigma : \Sigma \hookrightarrow \Sigma'$ is an inclusion, we only need to know

the names c_i of the symbols in Σ and the names h_j of the symbols in $\Sigma' \setminus \Sigma$, which are to be hidden. Then we can form N_{SP} by importing from $N_{SP'}$ and mapping all symbols that remain visible to their counterparts in N_{SP} and hiding the remaining symbols. The resulting diagram is the right diagram below. Note that by Lem. 4, $N_{SP}.t$ extends to a total $\mathbb{LF}$ morphism $N_{SP}.t^*$; moreover, it is easy to verify that $N_{SP}.t^*$ is an isomorphism.



5.3 Adequacy for Specifications

The general idea of the encoding of models is given in Fig. 3. The diagram corresponds to the one from Sect. 3.3 except that we have two extensions of L^{Syn}, namely N_Σ and N_{SP}. Consequently, we obtain two pushouts N_Σ^{Mod} and N_{SP}^{Mod} as shown in the left diagram. Then $(N_{SP}.s)^{mod}$ arises as the unique factorization through the pushout N_Σ^{Mod}.

Our central result will be that models $M \in Mod^I[SP] \subseteq Mod^I(\Sigma)$ can be represented as $\mathbb{LF}$ morphisms $m : N_\Sigma^{Mod} \to \mathcal{F}$ that factor through N_{SP}^{Mod}, i.e., such that there is an m^* with $m^* \circ (N_{SP}.s)^{mod} = m$.

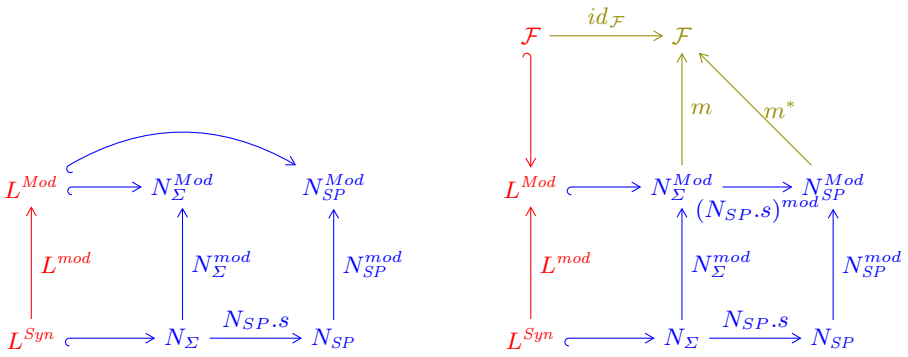
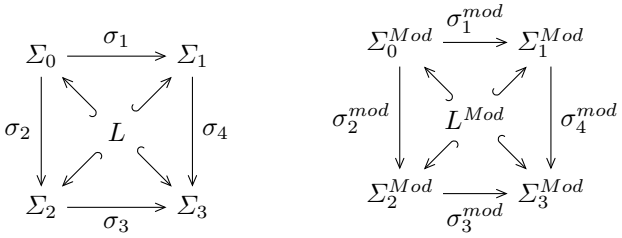


Fig. 3. Representation of Models in the Presence of Hiding

The translation of ASL to MMT yields pushouts between $\mathbb{LF}$ signatures extending L^{Syn}, but models are stated in terms of signatures extending L^{Mod}. Therefore, we use the following simple lemma:

Lemma 8. *Consider the left diagram below and assume that the rectangle is a pushout. For a morphism $L^{mod} : L \to L^{Mod}$, we form the signatures Σ_i^{Mod} as pushouts of Σ_i along L^{mod}; and we form the morphisms σ_i^{mod} as unique factorizations through the respective pushout. Then the rectangle in the resulting diagram on the right is also a pushout.*



Proof. This is shown with a straightforward diagram chase.

Then we are ready to state our main result:

Theorem 9. *Let I be an institution that is adequately represented in LF. Then for any signature Σ, any ASL-structured specification SP with $Sig[SP] = \Sigma$, and any Σ-model M*

$$M \in Mod^I[SP] \quad \text{iff} \quad \text{exists } m^* : N_{SP}^{Mod} \to \mathcal{F} \text{ such that } (N_{SP}.s)^{mod}; m^* = \ulcorner M \urcorner$$

Proof. The proof is done by induction on the structure of SP. All cases will refer to diagrams that correspond to those in Sect. 5.2, but which refer to the model theory rather than to the syntax – i.e., every node N_X becomes N_X^{Mod}. Lem. 8 ensures that when making this transformation, the pushout properties of the diagrams are kept.

Case $SP = (\Sigma, E)$**:**

For the base case, the conclusion follows directly from the assumption that the representation of I in LF is adequate.

Case $SP = SP_1 \cup SP_2$**:**

Let $M \in Mod[SP]$ and $m := \ulcorner M \urcorner : N_\Sigma^{Mod} \to \mathcal{F}$. We want to factor m through N_{SP}^{Mod}. By definition, we have that $M \in Mod[SP_1]$ and $M \in Mod[SP_2]$. By the induction hypothesis for SP_1 and SP_2, we get that there are morphisms $m_i : N_{SP_i}^{Mod} \to \mathcal{F}$ such that $m = (N_{SP_i}.s)^{mod}; m_i$. Using the pushout property we get a unique morphism $m^* : N_{SP}^{Mod} \to \mathcal{F}$ such that $(N_{SP_i}.s)^{mod}; (N_{SP'}.t_i)^{mod}; m^* = m$ which gives us the needed factorization.

For the reverse inclusion, let $m := \ulcorner M \urcorner : N_\Sigma^{Mod} \to \mathcal{F}$ represent a Σ-model M and factor as $(N_{SP}.s)^{mod}; m^*$. Notice that by composing $(N_{SP}.t_i)^{mod}$ with m^* we get morphisms $m_i : N_{SP_i}^{Mod} \to \mathcal{F}$. By using the induction hypothesis, M is then a model of both SP_1 and SP_2 and by definition M is a model of SP.

Case $SP' = \sigma(SP)$**:**

Let $M' \in Mod[SP']$ and $m' := \ulcorner M' \urcorner : N_{\Sigma'}^{Mod} \to \mathcal{F}$. We want to prove that there is $m'^* : N_{SP'}^{Mod} \to \mathcal{F}$ such that $m' = (N_{SP'}.s')^{mod}; m'^*$. By definition $M'|_\sigma \in$

$Mod[SP']$. By induction hypothesis for SP' we get a morphism $m := \ulcorner M'|_\sigma \urcorner : N_\Sigma^{Mod} \to \mathcal{F}$ and a morphism $m^* : N_{SP}^{Mod} \to \mathcal{F}$ such that $(N_{SP}.s)^{mod}; m^* = m = (N_\sigma)^{mod}; m'$, where the latter equality holds due to the definition of model reduct. Using the pushout property we get the desired m'^*.

For the reverse inclusion, assume $m' := \ulcorner M' \urcorner : N_{\Sigma'}^{Mod} \to \mathcal{F}$ that factors as $(N_{SP'}.s')^{mod}; m'^*$. Then $(N_{SP}.s)^{mod}; (N_{SP'}.t)^{mod}; m'^*$ factors through N_{SP}^{Mod} and thus by induction hypothesis the reduct of M' is an SP-model, which by definition means that M' is an SP'-model.

Case $SP = \sigma^{-1}(SP')$**:**

Let M be an SP-model and let $m := \ulcorner M \urcorner : N_\Sigma \to \mathcal{F}$. We want to prove that m factors through N_{SP}^{Mod}. By definition M has an expansion M' to an SP'-model. By induction hypothesis, there are morphisms $m' := \ulcorner M' \urcorner : N_{\Sigma'}^{Mod} \to \mathcal{F}$ and $m'^* : N_{SP'}^{Mod} \to \mathcal{F}$ such that $(N_{SP'}.s')^{mod}; m'^* = m'$. Then
$m = (N_{SP}.s)^{mod}; (N_{SP}.t^{*-1})^{mod}; m'^*$.

For the reverse inclusion, let $m := \ulcorner M \urcorner$ be a morphism that factors as $(N_{SP}.s)^{mod}; m^*$. We need to prove that M has an expansion to a SP'-model. We obtain it by applying the induction hypothesis to
$m' := (N_{SP'}.s')^{mod}; (N_{SP}.t^*)^{mod}; m^*$.

Corresponding to the adequacy for models, we can prove the adequacy for theorems by induction on SP, by observing that due to Lem. 4 structures always translate (possibly hidden) theorems to (possibly hidden) theorems.

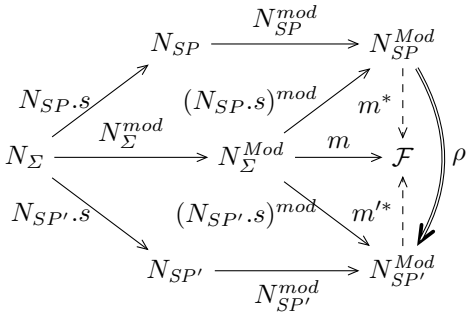
Theorem 10. *Let I be an institution and assume that I has been represented in LF in an adequate way. Then for any signature Σ, any ASL-structured specification SP with $Sig[SP] = \Sigma$, and any Σ-sentence F*

$$F \in Thm^I[SP] \quad \text{iff} \quad N_{SP}.s(l.\mathsf{ded}\ulcorner F \urcorner) \text{ inhabited over } N_{SP}$$

Note that both in Theorem 9 and Theorem 10 we make use of the fact that LF has model amalgamation for pushouts along injections. The results are thus valid also in the case when the institution I does not have model amalgamation or when the functor Φ of an adequate representation of I does not preserve pushouts.

5.4 Adequacy for Refinements

We want to give a syntactical criterion for refinement $SP \rightsquigarrow_\Sigma SP'$. Consider the diagram on the right. $SP \rightsquigarrow_\Sigma SP'$ states that for all m, if m'^* exists, then some m^* exists such that the diagram commutes. Clearly, this holds if there is an $\mathbb{LF}$ morphism $\rho : N_{SP}^{Mod} \to N_{SP'}^{Mod}$.



We can also prove the opposite implication if $\mathcal{F}$ has some additional technical properties. Intuitively, $\mathcal{F}$ must be able to represent I-models as $\mathcal{F}$-terms so that

we can switch back and forth between models represented as morphisms and models represented as terms. This is the case if $\mathcal{F}$ declares an operation of tupling so that the components of a morphism into $\mathcal{F}$ can be collected in a tuple.

Theorem 11. *Let I be an institution that is adequately encoded in LF. Moreover, assume that (i) $\mathcal{F}$ declares an operation of tupling and all I-models of a finite Σ can be represented as $\mathcal{F}$-tuples whose components correspond to the declarations in $\Sigma^{Mod} \setminus L^{Mod}$, and (ii) whenever $\mathcal{F}$ can prove the existence of such a tuple, there is an $\mathcal{F}$-term for such a model.*

Then for ASL-specifications SP and SP' over the signature Σ, we have that $SP \rightsquigarrow_\Sigma SP'$ iff there is an $\mathbb{LF}$ morphism $\rho : N_{SP}^{Mod} \to N_{SP'}^{Mod}$ such that $(N_{SP}.s)^{mod}; \rho = (N_{SP'}.s)^{mod}$.

Proof. The right-to-left implication follows immediately using Thm. 9.

For the left-to-right implication, we assume $SP \rightsquigarrow_\Sigma SP'$ and work within $N_{SP'}^{Mod}$. Due to the adequacy of $\mathcal{F}$, $\ulcorner SP \rightsquigarrow_\Sigma SP' \urcorner$ is a provable statement of $\mathcal{F}$ and thus of $N_{SP'}^{Mod}$ (iii). Using (i), we can tuple the declarations in $N_{SP'}^{Mod} \setminus \mathcal{F}$ (excluding the axioms) and obtain a Σ' model m as a term over $\mathcal{F}$; using the axioms in $N_{SP'}^{Mod} \setminus \mathcal{F}$, we can prove that m' is an SP' model. Using (iii), we show that m is also an SP model. Then using (ii), we obtain an expression m^* over $N_{SP'}^{Mod}$ that expresses a model of $N_{SP'}$. Finally, using (i), we can project out the components of m^*. The morphism ρ maps every symbol of $N_{SP}^{Mod} \setminus \mathcal{F}$ (excluding the axioms) to the corresponding components; it maps all axioms to proofs about m^*.

The assumption (i) of this theorem, while very specific, is mild in practice. In fact, if (i) did not hold, it would be dubious how the foundation can express institutions and models at all. The assumption (ii) is more restricting. For example, it does not hold necessarily for ZF set theory with a choice axiom but without a choice *operator*: without a choice operator, we may be able to prove the existence of a model in $\mathcal{F}$ without being able to turn it into a morphism into $\mathcal{F}$. Alternatively, we can establish (ii) for individual institutions by giving a constructive model existence proof.

6 Related Work

Our use of MMT is akin to the SML-style structure declarations that can occur within Extended ML signatures [KST97]. In fact, Extended ML can be extended with hiding in the same way as we extend LF+MMT, and a similar representation of ASL in Extended ML could be defined. The key advantage of LF+MMT is that LF is strong enough to encode to foundation as well and thus models as morphisms.

Our definition of hiding in LF+MMT is motivated by the approach taken in [GR04]. They also use a pair of two signatures one of which gives the visible interface. The flattening of LF+MMT with hiding corresponds to their semantics.

[BHK90] introduces *normal forms* for ASL-like structured specifications over many sorted classical first-order logic, where the normal form of a structured specification SP has the form $\sigma^{-1}(\Sigma, E)$, such that the normal form has the same model class as the original specification. This has been generalized to an arbitrary institution with weak amalgamation property in [Bor02]. The connection between our work and normal forms is done by the following result, that can be proved by structural induction.

Theorem 12. *Let SP be a structured specification and let us denote $nf(SP)$ its normal form. Then $N_{nf(SP)}$ is a flat H-signature that is isomorphic to the flattening of N_{SP}.*

The intuitive idea is that both the normal form and the hidden signature of N_{SP} hide the same symbols.

7 Conclusion

With the translation presented in this paper, it is possible to encode ASL- and CASL-style structured specifications with hiding in proof theoretic logical frameworks. This provides a new perspective on structured specifications that emphasizes constructive and mechanizable notions. Our translation is given for MMT-structured LF, but it easily generalizes to other MMT-structured logical frameworks.

Our work does not resolve the controversy between the formalistic (or proof-theoretic) and the semantic (or model-theoretic) approach. This, of course, could not be expected as certain aspects of a Platonic foundation are necessarily out of reach for a formalistic treatment. However, we have shown that substantial aspects of the semantic intuitions – here in particular, hiding – can be captured adequately in a formalistic framework.

Our encoding can be generalized to specifications represented as development graphs. In this context, our representation theorem for refinements can be strengthened to represent the hiding theorem links of [MAH06]. Even heterogeneous specifications [MT09,MML07] can be covered: as LF+MMT uses the same structuring operations for logics as for theories, this requires only the representation of the involved logics and logic translations in LF.

A theorem very similar to our representation theorem for refinements can be obtained for conservative extensions. This permits the interpretation of the proof calculus for refinement given in [Bor02]. In particular, the rules using an oracle for conservative extensions can be represented elegantly as the composition of LF signature morphisms.

The translation to MMT also has the benefit that we can re-use the infrastructure provided by languages like OMDoc [Koh06] (an XML-based markup format for mathematical documents) and tools like TNTBase [ZK09] (a versioned XML database for OMDoc documents that supports complex searches and queries, e.g., via XQuery). Further tools developed along these lines are

the JOBAD framework (a JavaScript library for interactive mathematical documents), which will provide a web-based frontend for the Heterogeneous Tool Set, GMoc (a change management system), DocTip (a document and tool integration platform) and integration with the Eclipse framework (an integrated development environment).

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