Task space controller for the novel Active Ankle

Shivesh Kumar, Bertold Bongardt, Marc Simonsfke, Frank Kirchner
Robotics Innovation Center (RIC), DFKI, Bremen, Germany

Introduction
Active Ankle is a novel parallel manipulator with three degrees of freedom that operates in an almost-spherical manner [1, 2]. The almost-spherical parallel manipulator (ASPM) is primarily intended as an actuated ankle joint in a full-body exoskeleton for rehabilitation application (Fig. 3).

Design features
1. lightweight and robust construction
2. modular design leading to low divergency
3. high stiffness and orientation accuracy
4. high payload capacity
5. no torque required for loads along torsional axis

Fig. 1: Active Ankle prototype
Fig. 2: Sketch of the Active Ankle
Fig. 3: Active Ankle with foot unit
Fig. 4: Scheme, \( r, d = 35 \), \( l = 100 \).

Control challenge
Due to spatial behaviour but spherical use case of the Active Ankle, the task space control of this mechanism asks for a joint configuration for a given orientation from \( SO(3) \), instead of a pose from \( SE(3) \) [3].

Inverse Geometric Model (IGM)
The Inverse Geometric Model (IGM) is a solution to the problem of finding input joint angles \( q_1, q_2, q_3 \) for a specific end-effector pose \( P_E = [s, n, a] \in SE(3) \), denoted as \( [q_1, q_2, q_3] = IGM(P_E) \).

Crank & endeffector points
The crank points \( (c_1, c_2, c_3, c_4) \) are allowed to move on the circles defined by the motion of three actuators. The end effector points \( (e_1, e_2, e_3, e_4, e_5) \) lie on a sphere of radius \( d \) and center \( e \).

The point parametrizations (CPL & EPL) are:
\[
\begin{align*}
\mathbf{c}_1 &= [0. r \cos(q_1), l + r \sin(q_1)]^T \\
\mathbf{c}_2 &= [0. r \cos(q_2), l - r \sin(q_2)]^T \\
\mathbf{c}_3 &= [(l + r \sin(q_3), 0. r \cos(q_3))]^T \\
\mathbf{c}_4 &= [(l - r \sin(q_3), 0. r \cos(q_3))]^T \\
\mathbf{c}_5 &= [(r \cos(q_4), l + r \sin(q_4))]^T \\
\mathbf{e}_1 &= e + d - n \\
\mathbf{e}_2 &= e + d - n \\
\mathbf{e}_3 &= e + d + s \\
\mathbf{e}_4 &= e + d - s \\
\mathbf{e}_5 &= e + d + a
\end{align*}
\]

RIGM solution
Equations (1) - (6) are highly coupled. A novel iterative algorithm has been developed which can be explained by the concept of virtual joints. The method RIGM implements a three-sphere intersection to solve for \( e \) [3].

Algorithm 1 Rotative Inverse Geometric Model (RIGM)

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \mathbf{P}<em>E \leftarrow [R_E \ 0</em>{0 \times 3}] ) ( \triangleright ) Initialization</td>
</tr>
<tr>
<td>2</td>
<td>while ( \mathbf{c} \neq \mathbf{P}_E ) do</td>
</tr>
<tr>
<td>4</td>
<td>( \mathbf{e}_1 \leftarrow \text{EPL}(\mathbf{P}_E) )</td>
</tr>
<tr>
<td>5</td>
<td>( \mathbf{e}_2 \leftarrow \text{IGM}(\mathbf{P}_E) )</td>
</tr>
<tr>
<td>6</td>
<td>( \mathbf{e}_3 \leftarrow \text{CPL}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3) )</td>
</tr>
<tr>
<td>7</td>
<td>( \mathbf{d} = \text{TFGM}(\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3, \mathbf{R}_E) ) ( \triangleright ) Error</td>
</tr>
<tr>
<td>8</td>
<td>( \mathbf{P}<em>E \leftarrow [R_E \ 0</em>{0 \times 3}] ) ( \triangleright ) Update</td>
</tr>
<tr>
<td>10</td>
<td>( [q_1, q_2, q_3] \leftarrow [\mathbf{q}_1, \mathbf{q}_2, \mathbf{q}_3] )</td>
</tr>
<tr>
<td>11</td>
<td>( [e_1, e_2, e_3] \leftarrow [\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3] )</td>
</tr>
<tr>
<td>12</td>
<td>return ( [q_1, q_2, q_3, e_1, e_2, e_3] )</td>
</tr>
</tbody>
</table>

Three virtual leg equations
By subtracting (2) from (1), (4) from (3), (6) from (5), three virtual leg equations are derived
\[
\begin{align*}
\mathbf{r}_E \cdot \cos(q_1) + r \cdot (e - l) \cdot \sin(q_1) + d \cdot (l - e + n) &= 0 \\
\mathbf{r}_E \cdot \cos(q_2) + r \cdot (e - l) \cdot \sin(q_2) + d \cdot (l - e + a) &= 0 \\
\mathbf{r}_E \cdot \cos(q_3) + r \cdot (e - l) \cdot \sin(q_3) + d \cdot (l - e + s) &= 0
\end{align*}
\]

With leg index \( i \in \{1, 2, 3\} \), they are of the form
\[
E_i \cdot \cos(q_i) + F_i \cdot \sin(q_i) + G_i = 0 \quad (7)
\]

IGM Solution
By tangent half angle substitution \( t = \tan(q_i/2) \), \( \cos(q_i) = (1 - t^2)/(1 + t^2) \), \( \sin(q_i) = 2t/(1 + t^2) \), the equation
\[
(G_i - E_i) \cdot t^2 + 2F_i \cdot t + (G_i + E_i) = 0
\]

in \( t \) is obtained. The two solutions for \( q_i \) are given by
\[
\begin{align*}
q_i &= 2 \cdot \arctan(2F_i \pm H_i, G_i - E_i) \\
H_i &= \sqrt{F_i^2 + F_i^2 - G_i^2}, \quad \text{see [3]}
\end{align*}
\]

Experimental results
RIGM has been implemented for a task space control of the Active Ankle. For \( e = 1.0 \times 10^{-6} \) mm, the algorithm can be realized at a control frequency of 10 kHz [3].

Conclusions
The novel Active Ankle mechanism is briefly presented along with relevant geometric and cinematic models for its control in task-space. In the future, the cascaded task-space control framework will be equipped with torque control.

Acknowledgment
The work presented in this paper was performed within the project Recupera-Raha, funded by the German Aerospace Center (DLR) with federal funds from the Federal Ministry of Education and Research (BMBF) (Grant 01IM14064A).

References

Fig. 5: Active joint angles during the dorsiflexion – plantarflexion motion.
Fig. 6: End effector shift during the dorsiflexion – plantarflexion motion.